Math Reading Test*

1. Does England have a fourth of July?

2. If you had only one match and entered a room where there was a lamp, an oil heater, and some kindling wood, which would you light first?

3. A woman gave a beggar 50 cents. The woman is the beggar's sister, but the beggar is not the woman's brother. Why?

4. Is it legal in North Carolina for a man to marry his widow's sister?

5. A garden has exactly 50 different kinds of flowers, including 10 kinds of roses, 3 kinds of sweet peas, 2 kinds of alyssum, 5 kinds of carnations, 3 kinds of zinnias, 8 kinds of poppies, 4 kinds of snapdragons, 5 kinds of gladiolas, and 6 kinds of phlox. How many different kinds of flowers did the garden have?

6. Abbott, Balzac, and Chang are a detective, an entomologist, and a farmer, although not necessarily in that order. Abbott was the proud mother of healthy twins yesterday. Chang has a deathly fear of insects and will not even get close enough to one to kill it if she sees it. The farmer is getting worried because she and her husband are getting old and will not be able to run the farm for too many more years, and she has no children. Chang, unmarried, especially likes to date brunettes. Match the women with their occupations.

7. Three quarrelsome people registered at a hotel and paid $30 for a suite of rooms, each person contributing $10. The clerk discovered later that he should have charged them only $25 for that suite, so he gave the bellboy $5 to return to the guests. Remembering how the guests had quarreled when they registered, the bellboy thought they would quarrel too much about how to split up the $5 refund, and so he kept $2 and returned $3 to the three people. Now each person had paid $10, less $1 refund for the room ($9 x 3 people = $27). But $27 plus the $2 kept by the bellboy equals only $29, and the original charge was $30. What happened to the other $1?

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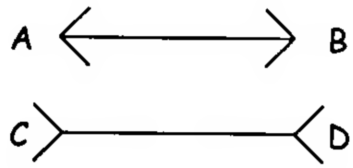
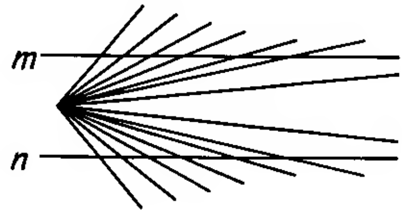
* From p. 34 of *Developing Math Learning Skills*, by Susan C. Brown, Margaret M. Scott, and Sandra Geiger of the Women's Educational Equity Act Program, New Mexico State University, Las Cruces, New Mexico. Adapted from Jean Smith, Math Clinic, Wesleyan University, Middletown, Connecticut.

Visual Puzzles*

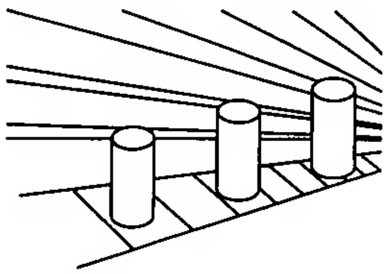
1. Is it further from A to B or from C to D?



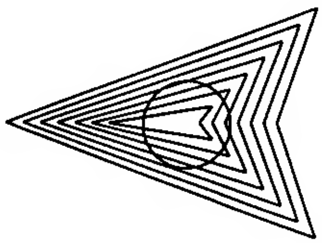
2. Would line *m* meet line *n* if they were extended?



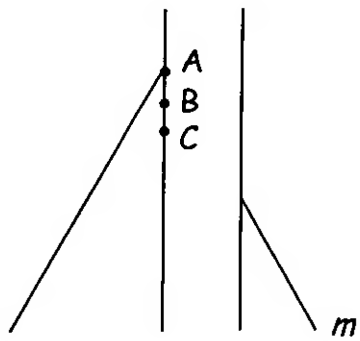
3. Which cylinder is tallest?



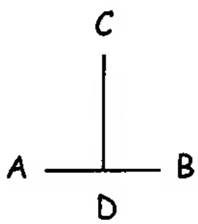
4. Is the figure in the center a perfect circle?



5. Will line *m*, when extended, meet point *A*, point B, point *C*, or none of these points?



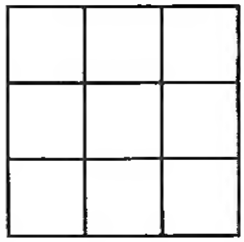
6. Which segment is longer, AB or CD?



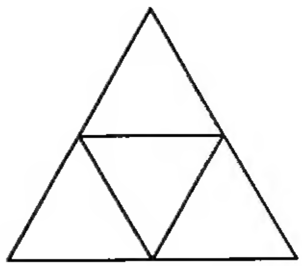
* From p. 9 of *Geometry: an Investigative Approach, 2nd Ed.*, by Phares G. O'Daffer and Stanley R. Clemens, Addison-Wesley Publishing Company, New York, 1992.

Toothpick Puzzles*

1. Use 24 toothpicks to make this figure.
   a) Remove 4 toothpicks and leave 5 squares.
   b) Remove 8 toothpicks and leave 4 squares.
   c) Remove 8 toothpicks and leave 2 squares.
   d) Remove 8 toothpicks and leave 3 squares.
   e) Remove 6 toothpicks and leave 3 squares.

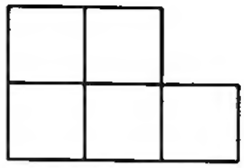


2. With 9 toothpicks, make this figure.
   a) Remove 2 toothpicks and leave 3 triangles.
   b) Remove 3 toothpicks and leave 1 triangle.
   c) Remove 6 toothpicks to get 1 triangle.
   d) Remove 4 toothpicks to get 2 triangles.
   e) Remove 2 toothpicks to get 2 triangles.

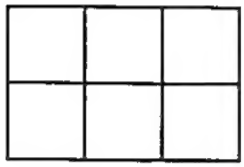


* From p. 38 of *Developing Math Learning Skills: A Parallel Support Course for the Math-Anxious College Student*, by Susan C. Brown, Margaret M. Scott, and Sandra Geiger of the Women's Educational Equity Act Program, New Mexico State University, Las Cruces, New Mexico. Adapted from handouts provided by the EQUALS Program at the Lawrence Hall of Science, U.C. Berkeley.

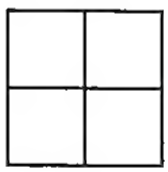
3. Use 15 toothpicks to make this figure.
   a) Remove 3 toothpicks and leave 3 squares.
   b) Remove 4 toothpicks and leave 3 squares.
   c) Remove 5 toothpicks and leave 3 squares.
   d) Remove 4 toothpicks and leave 2 squares.
   e) *Move* 3 toothpicks and make 4 squares.



4. Construct this figure using 17 toothpicks.
   a) Remove 5 toothpicks and leave 3 squares.
   b) Remove 6 toothpicks and leave 2 squares.

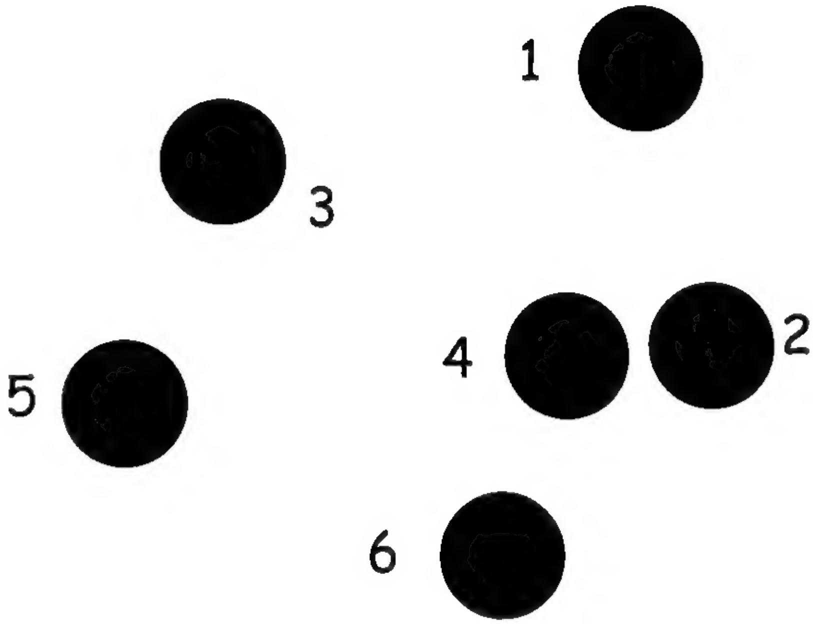


5. Use 12 toothpicks to make this figure.
   a) Remove 2 toothpicks to leave 2 squares.
   b) *Move* 4 toothpicks to make 10 squares.
   c) *Move* 2 toothpicks to make 7 squares.
   d) *Move* 3 toothpicks to make 3 squares.



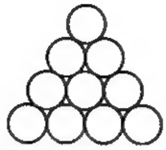
Dots

Move exactly one circle to another position so that their centers will be in four rows of three-in-a-line.



Some Fun Problems*

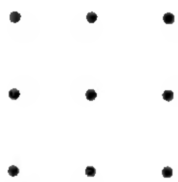
1. Move just three dots to form a triangle pointing down instead of up.



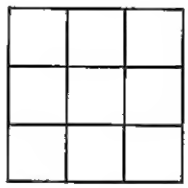
2. Form four equilateral triangles with just six toothpicks.

3. To mount a picture, two thumbtacks are needed in any two corners. What is the least number of tacks needed to mount four pictures? Include a picture.

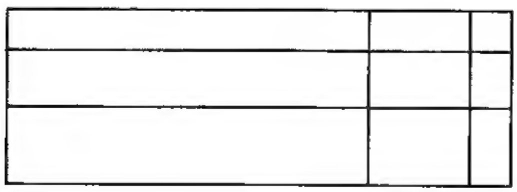
4. Without lifting your pencil from the paper or retracing any lines, try to draw four connected straight lines that pass through all nine points. Remember that the lines must be straight!



5. How many squares are in this figure? Explain.



6. How many rectangles are in this figure? Explain.



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* Problems 1 and 4 are from *Faces of Mathematics, 3rd Ed.*, by A. Wayne Roberts, HarperCollins College Publishers, 1995, pp. 9,41. Problems 2 and/or 3 are from p. 39 of *Teaching Mathematics* by Eric A. Sobel and Evan M. Maletsky, Allen and Bacon, 1991.

Tricky Puzzles

The following puzzles are warm-up problems. They have been around in one form or another for many years. They will help you begin to think and to understand what is really being asked in a problem.

1. How much dirt is in a hole 2 feet long, 3 feet wide, and 2 feet deep?

2. Two U.S. coins have a total value of 55 cents. One coin is not a nickel. What two coins are they?

3. A farmer went to sleep at 8:00 p.m. He set his alarm for 9:00 a.m. the next morning. For how many hours did he sleep before he was awakened?

4. Divide 30 by $\frac{1}{2}$ and then add 12 (don't use your calculator). What is the result?

5. Cesar owns 20 blue socks and 15 brown ones. What is the minimum number of socks which he must pull out of his drawer on a dark morning to be sure that he has a matching pair?

6. A heavy smoker wakes up in the middle of the night and finds herself out of cigarettes. This stores are closed, so she looks through all of the ashtrays for butts. (Can you believe this? Yuk!) She knows from past experience that she can make one cigarette from five butts. She finds 25 butts and decides they will last her until morning if she smokes only one cigarette every hour. For how many hours does her supply last?

7. You have eight sticks. Four of them are exactly one-half the length of the others. Enclose three equal squares with them.

8. Suppose you have only one 5-liter container and one 3-liter container. How can you measure exactly 4 liters of water if neither container is marked for measuring? (You have no other containers to use for this project).

9. It takes 1 hour 20 minutes to drive to the airport, yet the return trip takes just 80 minutes using the same route and driving at what would appear to be the same speed. How can this be?

10. Consider the following banking transactions. Deposit $50 and withdraw as follows:

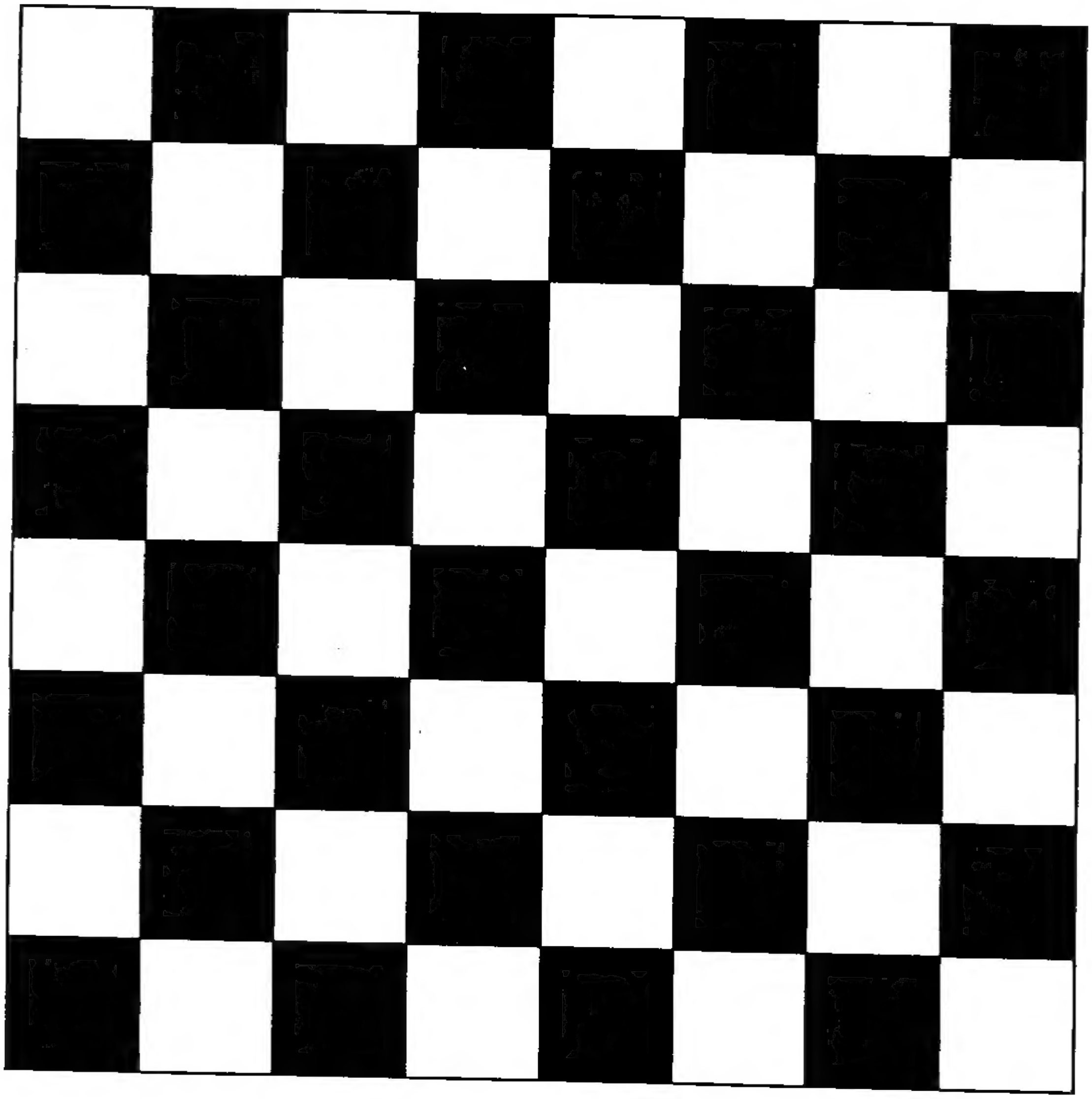
withdraw	$20.00	leaving	$30.00
withdraw	15.00	leaving	15.00
withdraw	9.00	leaving	6.00
withdraw	6.00	leaving	0.00
	$50.00		$51.00

Where did the extra dollar come from?

11. Come up with a "tricky puzzle" of your own, and share it with another group.

How Many Squares?

How many squares of any and all sizes are there on the checkerboard below? (There will be a lot of overlap.)



More Team Building Activities

Penny's Dimes

Penny has 25 dimes. She likes to put them in three piles with an odd number of dimes in each pile. In how many ways could she do this?

Penny's Dimes - Part II

Penny has spent some of her dimes and acquired some more. Although she knows that she has fewer than 100 dimes, she does not know the exact number. One day she was arranging the dimes on her desk in different ways. She found that when she placed them into piles of two, there was one left over. When she put them into piles of three, there was one left over. She them tried putting them into piles of five and found that there were none left over. How many dimes does Penny have?

Dodger Stadium

There is a joke among radio broadcasters about the number of people who start leaving Dodger Stadium in the seventh inning of baseball games. One evening during a particularly boring baseball game in which the Dodgers were trailing by six runs after six innings, the fans began to leave at a record pace. After the first out in the top of the seventh inning, 100 fans left. After the second out, 150 more fans left. After the third out, 200 more fans left. Fifty more fans left after each out than had left after the previous out. The ridiculous thing was that the Dodgers tied up the game in the bottom of the ninth inning and people still kept leaving early. The game lasted ten innings with the Dodgers finally losing in the tenth inning. How many fans left early?

Odd and Even

Find the difference between the sum of the first 500 even numbers and the sum of the first 500 odd numbers.

Fencing

A farmer buys 1000 yards of chain-link fencing. What is the largest rectangular area he can enclose with this amount of fencing?

A contractor was preparing an estimate for some work on Mr. Allen's lot. She asked Mr. Allen for the dimensions, but Mr. Allen had a poor memory and could not recall exactly. "I do remember," Mr. Allen told her, "that the lot is rectangular, and that it took 90 yards of fencing to enclose it." Determine possible dimensions for this lot.

The Great Train Robbery*

In London there were three gangs operating on August 11, 1891. Holmes knew from some inside information that his equal in crime, the clever Moriarty, led a gang with six members. At the same time the treacherous Smerzi headed a gang with seven members and Gilda Z., the trickiest of them all, a gang with eight members.

Watson: Have you figured out whose gang pulled off the Great Train Robbery of October 3, 1891?

Holmes: Yes.

Watson: But how, Holmes?!!

Holmes: Elementary arithmetic, my dear Watson.

Watson: Let me in on how you did it.

Holmes: From certain information in Scotland Yard, it was known that originally none of the gangs was large enough to pull off the Great Train Robbery. They must have added another organization that was twice the size of the original gang. Altogether, twenty-one members were involved in the robbery.

Watson: This is all too much for me. I hate math. I can't do it. I block and get anxious. Just tell me whose gang did it, I can't figure it out.

Whose gang pulled off the Great Train Robbery?

---

* Adapted from *Mind Over Math* by Kogelman & Warren, McGraw-Hill Book Co., 1978

Guessing[*]

"In order to make a discovery, it is necessary to make a guess. These guesses then need to be followed by verification." (George Polya)

- How long would it take to spend one million dollars at the rate of one hundred dollars every minute?

- A person starts a chain letter. The letter is sent to 2 people and each of them is asked in turn to send copies to 2 other people. These recipients are asked to send copies to 2 additional people. Assuming no duplication, how many people in all will have received copies of the letter after the twentieth mailing?

- Approximately how many pennies are there in one pound of pennies: 100, 150, 200, or 300?

- About how many pennies would have to be piled one on top of another to reach the ceiling of a room that is 8 feet high?

Bonus: Approximately how many hairs are there on your head? Be able to explain how you came up with your guess!

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[*] From p. 50 of *Teaching Mathematics* by Eric A. Sobel and Evan M. Maletsky, Allen and Bacon, 1991. The bonus was added by H. A. Lewis.

Those Old Math Problems

These problems are all over a thousand years old! They come from a Latin collection entitled *Problems for the Quickening of the Mind* that was written around the year 775.

Bushels

One hundred bushels of corn are to be divided among 100 people. Men get 3 bushels each, women get 2 bushels each, and children get $\frac{1}{2}$ bushel each. How many each of men, women, and children are there?

Flasks

Thirty flasks of wine - ten full, ten half-full, and ten empty - are to be divided among three sons so that each son gets the same number of flasks, and the same amount of wine. How can this be done without pouring?

Rivers

A wolf, a goat, and a cabbage must be moved across a river. The boat can only hold the ferryman and one other object. How can the ferryman move everything across the river so that the goat shall not eat the cabbage, and the wolf shall not eat the goat?

A modern variation of the river problem is found below:

Crossing the River

Nine men and two boys want to cross a river using an inflatable raft that will carry either one man or the two boys. How many times must the raft cross the river in order to accomplish this goal? (A round trip equals two crossings).

Choose a Number

1. Magic numbers!
   a) Pick your favorite number (any number).
   b) Multiply it by 25.
   c) Add 10.
   d) Multiply it by 6.
   e) Subtract 30.
   f) Divide by 15.
   g) Subtract 2.
   h) Divide by 10.
   i) You have your number back!

   Can you figure out why this works?

2. This one is more complicated, and it will only work in the year 2006!
   a) Pick the number of days a week that you would like to go out.
   b) Multiply this number by 2.
   c) Add 5.
   d) Multiply it by 50.
   e) If you have already had your birthday this calendar year, add 1756. If you have not had a birthday this calendar year, add 1755.
   f) Last step. Subtract the four-digit year that you were born (e.g. 1988).
   g) You should have a three-digit number. The first digit of this was your original number (i.e. how many times you want to go out a week). The second two digits are your age!!!

   Can you figure out why this works? [Hint: If you play this in 2007, add 1757/1756 in step (e). If you play this in 2008, add 1758/1757 in (e).]

Creepy Crawlies*

Rochelle collects lizards, beetles, and worms. She has more worms than lizards and beetles together. Altogether in the collection there are twelve heads and twenty-six legs. How many lizards does Rochelle have?

How Many Geese?**

In a Russian story, a lone goose was flying in the opposite direction from a flock of geese. He cried, "Hello, one hundred geese!" The leader of the flock answered, "We aren't one hundred! If you take twice our number and half our number, and add a quarter of our number, and finally add you, the result is one hundred."

How many geese are in the flock?

Egyptian Mathematics

The Rhind Papyrus is a collection of Egyptian mathematics problems, together with their solutions. It was written in about 1650 B.C.E. by a scribe named Ahmes, who claimed that this was a copy of an earlier work from the Twelfth Dynasty (1849 - 1801 B.C.E). Can you solve these problems?

1. A quantity and its 1/2 added together become 16. What is the quantity? (Problem 25 in the Rhind Papyrus)

2. A quantity and its 1/4 added together become 15. What is the quantity? (Problem 26 in the Rhind Papyrus)

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* From p. 44 of *Thinking Mathematically* by James Mason with Leone Burton and Kaye Stacey, Addison-Wesley Publishers.

** From p. 84 of *Faces of Mathematics, 3rd Ed.*, by A. Wayne Roberts, HarperCollins College Publishers, 1995

Time to Weigh the Hippos*

Martha is the chief hippopotamus caretaker at the Wild Animal Park in San Diego, California. She has just arrived at the cargo dock in order to pick up four members of the endangered species *hippopotamus mathematicus* that were recently rescued from African poachers. Before the animals can be released by the harbormaster, Martha must weigh them. Unfortunately, the only scale big enough to weigh a hippo is the truck scale that doesn't weigh anything lighter than 300 kilograms (kg); this is more than each of the hippos weighs. Martha is puzzled for a few minutes, then gets the idea of weighing the hippos in pairs, thinking that if she gets the mass of every possible pair, she can later figure out the masses of the individual hippos. She weighs the hippos pair-by-pair and gets 312 kg, 356 kg, 378 kg, 444 kg, and 466 kg. When she tries to weigh the heaviest pair of hippos, the scale breaks. Alas!

1. What was the mass of the last pair of hippos who broke the scale?

2. What are the masses of the individual hippos?

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* From the Math 130 course packet at the University of Wisconsin at Madison.

Painting the Lamp Posts*

Tim Murphy and Deborah Stern were engaged by the local authorities to paint the lamp posts on a certain street. Tim, who was an early riser, arrived first on the job. He had already painted three lamp posts on the south side of the street when Deb arrived, only to inform Tim that the contract stated that he was supposed to paint the lamp posts on the north side of the street. So Tim started afresh on the north side and Deb continued on the south side. When Deb finished on the south side, she went across the street and painted six posts for Tim on the north side and then the job was finished. If there is an equal number of lamp posts on each side of the street, the question is:

Who painted more lamp posts and just how many more?

* This problem has been passed around a bit, but the original source is unknown.

An Inductive Reasoning Game*

Symbols used: O + ∴ *

A

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

B

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

C

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

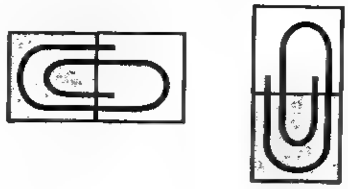
D

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

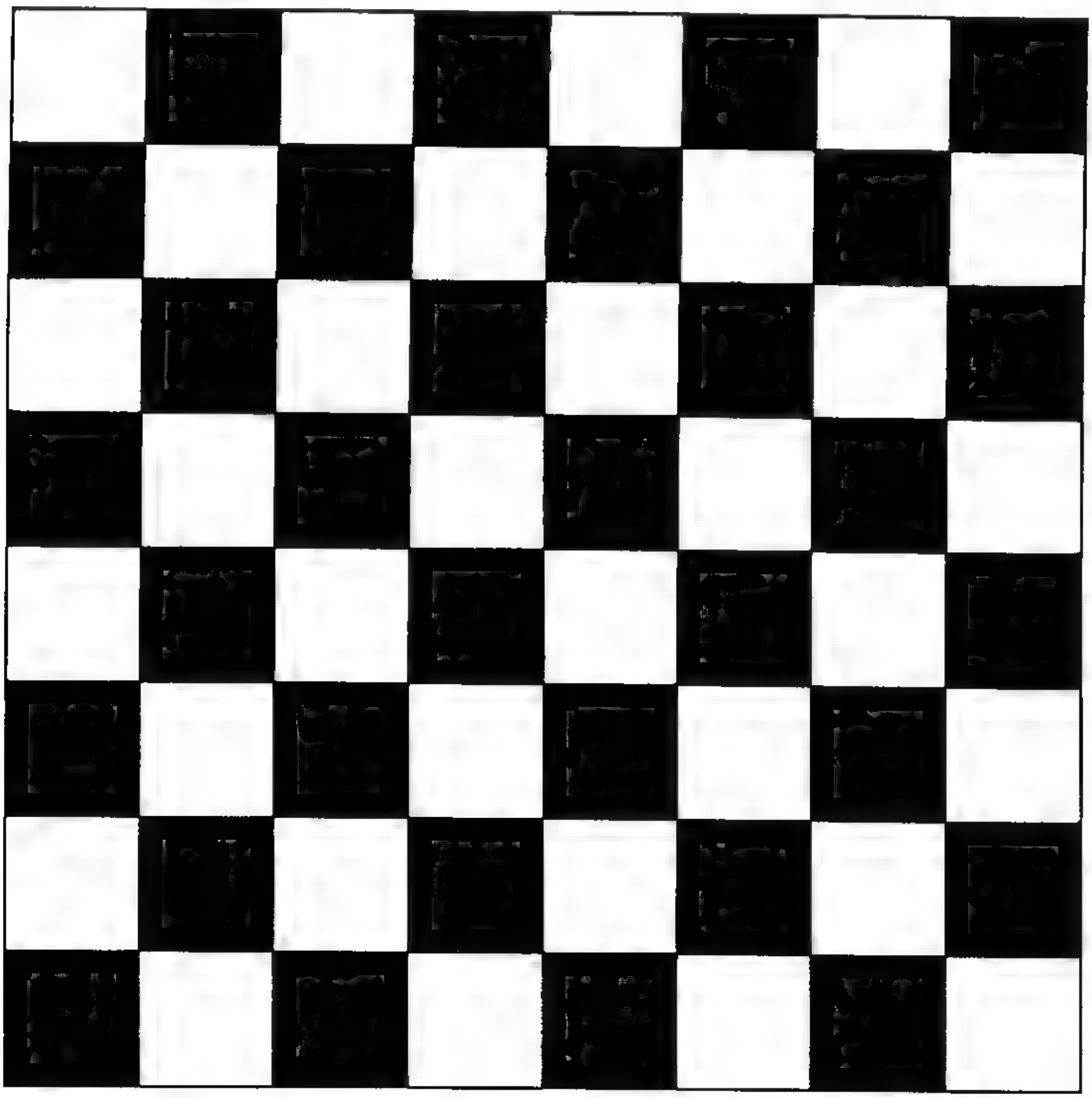
* From *Mathematics: A Human Endeavor, 2nd Ed.* by Harold R. Jacobs, W. H. Freeman and Company, 1982.

Covering the Board

1. Suppose you remove all four corners of the checkerboard. Can you cover all the squares in the checkerboard with paperclips? The paperclips should either be horizontal or vertical, and should cover exactly two squares at a time.

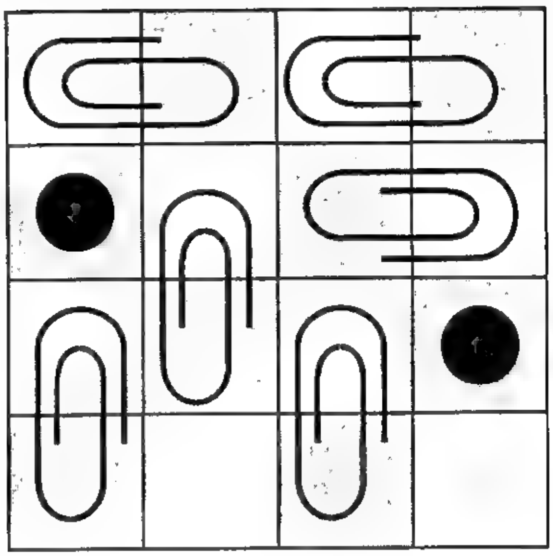


2. Suppose you remove the two white squares in the corners of the checkerboard. Can you cover the remaining squares with paperclips?

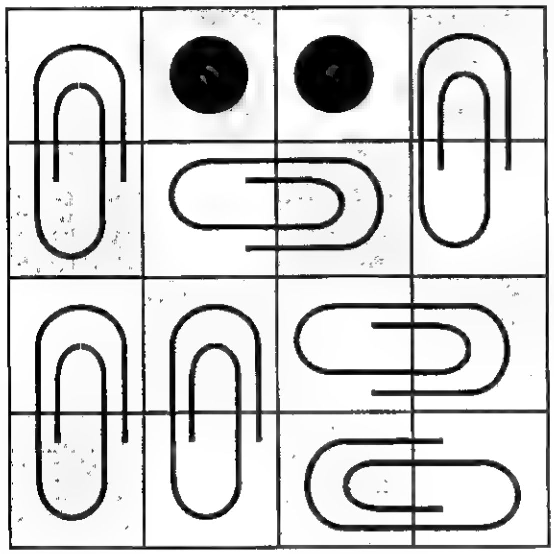


The Checkerboard Game*

Start with a small checkerboard made from sixteen squares. The first player puts pennies on any two of the squares. The second player then puts paperclips on the board so that each clip lies on two squares that are next to each other. The clips cannot overlap. To win, the second player has to put seven paperclips in this way so that they cover the fourteen squares that don't have pennies on them. If the second player can't do this, the first player wins. Two example are below.



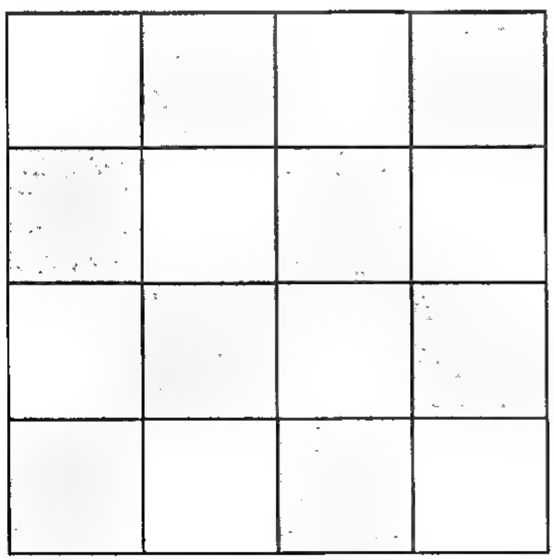
First player wins



Second player wins

Play the game a few times, until you have figured out whether you want to be the first player or second player, and how you want to play. After you have come up with a winning strategy, see if you can explain why it will always work.

Discovering a winning strategy through trial and error is an example of *inductive reasoning*. Explaining why your strategy will always work is an example of *deductive reasoning*.



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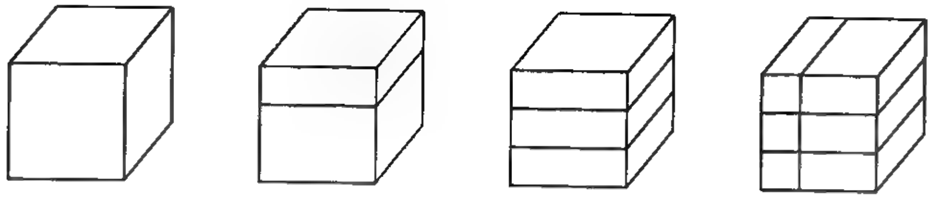
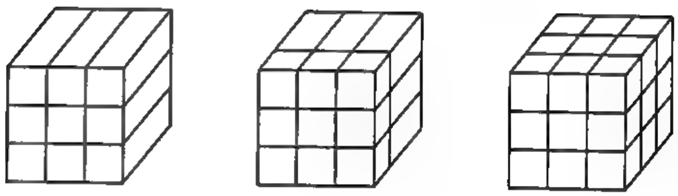
* From p. 33 of *Mathematics: A Human Endeavor, 2nd Ed.* by Harold R. Jacobs, W. H. Freeman and Company, 1982.

Sawing the Cube*

If you have a large cube of wood, you can make 27 smaller cubes by making six cuts in the wood. Notice that you don't have to move the wood at all.

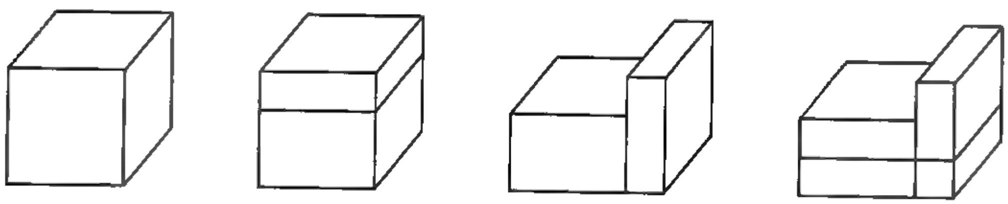


0 cuts 1 cut 2 cuts 3 cuts



4 cuts 5 cuts 6 cuts

Suppose you can move the pieces after each cut, as in the figures below.



0 cuts 1 cut move pieces 2 cuts

By moving the pieces around, you can cut more wood with the second cut. Can you make 27 smaller cubes in less than six cuts? What is the smallest number of cuts that you need?

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* From p. 48 of *Mathematics: A Human Endeavor, 2nd Ed.* by Harold R. Jacobs, W. H. Freeman and Company, 1982.

Painting Cubes

1. You put twenty-seven unit cubes together to make one large cube, and then paint all of the faces of the large cube.

   a) How many unit cubes have no paint on them?

   b) How many unit cubes have paint on exactly one face?

   c) How many unit cubes have paint on exactly two faces?

   d) How many unit cubes have paint on exactly three faces?

   e) How many unit cubes have paint on exactly four faces?

2. Now suppose you only paint two of the faces of the large cube. How many little cubes will have no paint on them?

3. What if you made a cube out of 1000 small cubes, and painted all six sides. How many unit cubes would have no paint on them? How many would have paint on exactly one face? On exactly two faces? On exactly three faces? On exactly four faces?

The Game of Poison*

Number of Players: Two teams of one or more players.

Equipment: Counters.

Rules:
1. Begin with seven counters.
2. Decide which team goes first.
3. When it is your team's turn, take away 1 or 2 counters.
4. The team that takes the last (POISONED) counter loses.

Play the game a few times. As you play, remember or keep a record of the game: How many counters did you pick up? Why? Who won the game?

After you have played the game a few times, reflect on your game records and see if your group can come up with a way to win. Do you want to go first or second? How many counters should you pick up? Once you have an idea, test it out against other players.

Once you have a strategy that works for seven counters, play the game with eight counters. Can you find a way to win? What about with nine counters? Ten? Eleven? Is there a pattern?

Rat Poison

To win you have to be as sneaky as a rat!

Rat Poison has the same rules as Poison except that in Rat Poison you may take 1, 3, or 4 counters; you *cannot* take 2 counters. The team that takes the last counter still loses.

Can you find a way to win if you start with seven counters? What about with eight? Nine? More?

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* Poison and Rat Poison are from the Math 130 course packet at the University of Wisconsin at Madison.

Sing it Again, Sam!

Country-Western songs emphasize three basic themes: love, prison, and trucks. A survey of the local Country-Western radio station produced the following data:

12	songs about a truck driver who is in love while in prison
13	songs about a prisoner in love
18	songs about a truck driver in love
28	songs *only* about being in love
3	songs about a truck driver in prison who is not in love
2	songs abut people in prison who are not in love and do not drive trucks
8	songs about people who are not in prison, are not in love, and do not drive trucks
16	songs about truck drivers who are not in prison

How many songs were surveyed?

Banned Book Survey

According to the American Library Association, three of the most frequently challenged novels in 1997 were *I know Why the Caged Bird Sings* by Maya Angelou, *The Adventures of Huckleberry Finn* by Mark Twain, and *The Giver* by Lois Lowry. As part of the Banned Books Week commemorations, the Nazareth Library surveyed 100 people on what previously banned books they had read. They found that 53 had read *Huckleberry Finn*; 42 had read *I know Why the Caged Bird Sings*, and 36 had read *The Giver*. To get more detailed information, the library staff did a follow-up survey and found that 25 of the people had read both *Huckleberry Finn* and *I know Why the Caged Bird Sings*, while 14 had read *I know Why the Caged Bird Sings* and *The Giver*, and only 10 had read *Huckleberry Finn* and *The Giver*. A reporter tried to figure out from these data how many of the people surveyed had read all three books. It turned out that 12 people surveyed had not read any of the books. Did the reporter need this last piece of information to answer her question? How many of the people had read all of the books?

Bixley to Quixley*

Here is a pretty problem which was figured out during Marco's ride from Bixley to Quixley atop a razor back mule. Marco was be traveling with a guide, Don Pedro, and it is assumed they were traveling at a constant speed. The journey would begin in Bixley and the two would be passing through Pixley on their journey to get some liquid refreshments, and then end as indicated in Quixley. After the two had been traveling for forty minutes, Marco asked Don Pedro how far they had gone, and Don Pedro replied, "Just half as far as it is from here to Pixley."

After creeping along for another seven miles, Marco asked, "How far is it to Quixley?" Don Pedro replied as before, "Just half as far as it is from here to Pixley." The two arrived at Quixley in another hour. It is your job to determine the distance from Bixley to Quixley.

* This problem has been passed around a bit, but the original source is unknown.

Word Sums*

1.

a) In the sum on the right, the letters stand for numbers. Each of the letters F, O, U, R, N, E, I, V stands for one of the digits 0 – 9. Different letters stand for different digits, and the first digit of a number is never zero (so the letters F and O cannot be zero). Can you replace the letters by digits to get a correct sum?

$$\begin{array}{r} \text{F O U R} \\ +\ \text{O N E} \\ \hline \text{F I V E} \end{array}$$

b) Here are two more word-sums like the ones in part a). Can you replace the letters by numbers to get a correct sum?

$$\begin{array}{r} \text{O N E} \\ +\ \text{O N E} \\ \hline \text{T W O} \end{array} \qquad\qquad \begin{array}{r} \text{T W O} \\ +\ \text{T W O} \\ \hline \text{F O U R} \end{array}$$

The first one has sixteen different solutions. See how many you can find! The second one has eight different solutions. How many can you find?

2. Many word-sums look promising but don't actually work! When you come to have a go at harder problems (like the ones in Question 4) it is important to be able to choose a good place to begin. One way of learning to "spot" good starting points is to try to explain very simply why certain word-sums can't be done. Most of the word-sums in this question can't be done. Find the ones which can't be done and explain why they are impossible.

$$\begin{array}{r} \text{F I V E} \\ +\ \text{T W O} \\ \hline \text{S E V E N} \end{array} \qquad \begin{array}{r} \text{T H R E E} \\ +\ \text{N I N E} \\ \hline \text{T W E L V E} \end{array} \qquad \begin{array}{r} \text{F O R T Y} \\ +\ \text{F O R T Y} \\ \hline \text{E I G H T Y} \end{array}$$

$$\begin{array}{r} \text{E I G H T} \\ +\ \text{F O U R} \\ \hline \text{T W E L V E} \end{array} \qquad \begin{array}{r} \text{E L E V E N} \\ +\ \text{O N E} \\ \hline \text{T W E L V E} \end{array} \qquad \begin{array}{r} \text{F O U R} \\ +\ \text{F I V E} \\ \hline \text{N I N E} \end{array}$$

$$\begin{array}{r} \text{T W E L V E} \\ +\ \text{E I G H T} \\ \hline \text{T W E N T Y} \end{array} \qquad \begin{array}{r} \text{T H R E E} \\ +\ \text{F I V E} \\ \hline \text{E I G H T} \end{array} \qquad \begin{array}{r} \text{T H I S} \\ +\ \text{O N E 'S} \\ \hline \text{A W F U L} \end{array}$$

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* From pp. 36-37 of *Mathematical Puzzling* by A. Gardiner, Oxford University Press, 1993.

3. Can you work out what T must be in this one?

$$\begin{array}{r} \text{T H I S} \\ +\quad \text{I S} \\ \hline \text{H A R D} \end{array}$$

4. The most satisfying word-sums are those that can be shown to have just one solution by elementary reasoning. Most of the word-sums in this question are of this kind. They are harder, but more interesting, than the previous ones.

   a) How many different solutions are there to this one?

$$\begin{array}{r} \text{H O C U S} \\ +\ \text{P O C U S} \\ \hline \text{P R E S T O} \end{array}$$

   b) Find all possible solutions to each of these:

$$\begin{array}{r} \text{S C A N} \\ +\ \text{T H E S E} \\ \hline \text{D I G I T S} \end{array} \qquad \begin{array}{r} \text{C R O S S} \\ +\ \text{R O A D S} \\ \hline \text{D A N G E R} \end{array} \qquad \begin{array}{r} \text{B E E R} \\ +\ \text{B E E R} \\ \hline \text{D R U N K} \end{array}$$

$$\begin{array}{r} \text{T E N} \\ \text{T E N} \\ +\ \text{F O R T Y} \\ \hline \text{S I X T Y} \end{array} \qquad \begin{array}{r} \text{S E V E N} \\ \text{S E V E N} \\ +\quad \text{S I X} \\ \hline \text{T W E N T Y} \end{array} \qquad \begin{array}{r} \text{S E V E N} \\ \text{T H R E E} \\ +\quad \text{T W O} \\ \hline \text{T W E L V E} \end{array}$$

5. How many different solutions are there to this one? (Carl Friedrich Gauss was perhaps the greatest mathematician who every lived. He was born in the year 1777 in the town of Braunschweig, and died in 1855 in Göttingen.)

$$\begin{array}{r} 1\ 7\ 7\ 7 \\ 1\ 8\ 5\ 5 \\ +\ \text{C A R L} \\ \hline \text{G A U S S} \end{array}$$

6. To end up, here are three with a seasonal flavor. The first has just one solution, the second has two solutions, and the third has four solutions!

$$\begin{array}{r} \text{A} \\ \text{M E R R Y} \\ +\quad \text{X M A S} \\ \hline \text{T U R K E Y} \end{array} \qquad \begin{array}{r} \text{X M A S} \\ \text{A}\overline{)\text{H A P P Y}} \end{array} \qquad \begin{array}{r} \text{X M A S} \\ \text{A}\overline{)\text{M E R R Y}} \end{array}$$

Are You a Genius?*

Each problem is a series of some sort - that is, a succession of letters, numbers, or drawings - with the last item in the series missing. Each series arranged according to a different rule and, in order to identify the missing item, you must figure out what the rule is. If you enjoy testing yourself, see how far you can get in 20 minutes.

In the problems done with drawings, it is always the leftmost row or group that needs to be completed by choosing one drawing from the row on the right.

Problem 1: A D G J

Problem 2: 1 3 6 10

Problem 3: 1 1 2 3 5

Problem 4: 21 20 18 15 11

Problem 5: 8 6 7 5 6 4

Problem 6: 40 35 34 29 28 23

Problem 7:

Problem 8:

Problem 9:

Problem 10:

Problem 11:

---

* Adapted from a MENSA test.

Problem 12: 

Problem 13: 

Problem 14: 

Can you make your own?

Problem 16: 

Problem 17: 

Problem 18: 

More Problems for You!

The Professor's Question

On a 24-item true-false test, a professor gave 5 points for each correct response but took off 7 points for each wrong one. Andy answered all of the questions and came up with a big fat zero for his score. How many did he get right?

Buying Wine

This problem appears in *Zhang Quijian Suanjing* (*Zhang Qiujian's Mathematical Manual*), which was written in China in the year 475.

> Each pint of high-quality wine costs 7 coins, each pint of ordinary wine costs 3 coins, and 3 pints of wine dregs cost 1 coin. If 10 coins are used to buy 10 pints in all, find the amount of each type and the total money spent on each.

Ladies Luncheon*

Five women have lunch together seated around a circular table. Ms. Harris is sitting between Ms. Lee and Ms. Martin. Ellen is sitting between Jessica and Ms. Norris. Ms. Lee is between Ellen and Mina. Jessica and Leticia are sisters. Ilse is seated with Ms. Lopez on her left and Ms. Martin on her right. Match the first names with the surnames.

Building a Table

The Procrastinator's Club is planning a party for its loyal members. Twenty-two people have said they will attend. Card tables which seat one person on each side will be placed in a rectangular shape so that when the tables are covered with tablecloths, they will look like one large table. How can the tables be arranged?

---

* From p. 32 of *Thinking Mathematically* by James Mason with Leone Burton and Kaye Stacey, Addison-Wesley Publishers

What is the Score?

In one week, three hockey teams, The Islanders, The Rangers, and The Sabres, played each other once with no shutouts resulting. Teams get 10 points for a win, 5 points for a tie, and 1 point for each goal scored. The Islanders got 20 points during the week, the Rangers got 19 points, and the Sabres got 5 points. What were the results of these games?

First in, Last Out*

In her office, Ms. Nguyen will at various times of the day drop a letter to be typed in her secretary's tray. She numbers these letters as she drops them in. When there is time, the secretary takes the top letter from her basket and types it. On a day when six letters got dropped in and typed, which of the following could *not* be the order in which the secretary typed them?

a) 546321 b) 425631 c) 354261 d) 246531

* Adapted from p. 75 of *Faces of Mathematics, 3rd Ed.*, by A. Wayne Roberts, HarperCollins College Publishers, 1995.

The Great Train Robbery*

In London there were three gangs operating on August 11, 1891. Holmes knew from some inside information that his equal in crime, the clever Moriarty, led a gang with six members. At the same time the treacherous Smerzi headed a gang with seven members and Gilda Z., the trickiest of them all, a gang with eight members.

Watson: Have you figured out whose gang pulled off the Great Train Robbery of October 3, 1891?

Holmes: Yes.

Watson: But how, Holmes?!!

Holmes: Elementary arithmetic, my dear Watson.

Watson: Let me in on how you did it.

Holmes: From certain information in Scotland Yard, it was known that originally none of the gangs was large enough to pull off the Great Train Robbery. They must have added another organization that was twice the size of the original gang. Altogether, twenty-one members were involved in the robbery.

Watson: This is all too much for me. I hate math. I can't do it. I block and get anxious. Just tell me whose gang did it, I can't figure it out.

Whose gang pulled off the Great Train Robbery?

* Adapted from *Mind Over Math* by Kogelman & Warren, McGraw-Hill Book Co., 1978

Experiments*

Experiment 1

Material: One sheet of paper.

Directions: Fold the piece of paper once, open it up, and record the number of regions.

Close the paper (along the first fold) and fold a second time, trying for the maximum number of regions possible. Record the number of regions there are when you open it up.

Close the paper (along the first two folds) and fold it a third time. How many regions are there now?

Try to discover the number sequence and predict the result for four, five, and six folds. Check your answers.

How many regions are there if you fold the paper n folds.

---

* Adapted from p. 116 of *Thinking Mathematically* by James Mason with Leone Burton and Kaye Stacey, Addison-Wesley Publishers.

Experiment 2

Material: String and scissors.

Situation I: If you know the number of cuts in a string, do you know the number of pieces?

If there are c cuts in the string, how many pieces will there be? If there are p pieces, how many cuts were made?

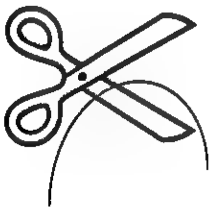
Situation II: Fold the string in half and cut the string once (not at the fold). How many pieces are there?

Take a new string, fold it in half, and cut it twice (but not at the fold). How many pieces are there?

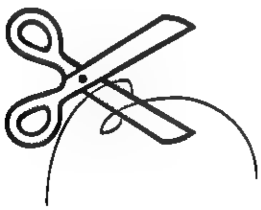
Try to find a pattern: if you take a fresh string, fold it in half, and cut it c times, how many pieces will there be?

If someone else in your group takes a fresh string, cuts it a few times, and ends up with p pieces, can you figure out how many times they cut it?

Situation III: Loop the string through the scissors as shown.



0 loops



1 loop

Cut the string once. Determine the number of pieces formed from a given number of loops.

If there are n loops in the string, how many pieces will there be? If there are p pieces, how many loops were made?

Chain Letter

This is an ACTUAL LETTER that was sent to one of our mathematics faculty. If everyone followed the instructions, how much money (exactly) would you get?

My name is Nancy Gorman. In September of 1983 my car was repossessed and bill collectors were hounding like you wouldn't believe. I was laid off and unemployment ran out. In January 1984 my family and I went on a 10 day cruise, and in February 1984 I bought a 1984 Cadillac with cash.

I am currently building a home in Virginia and I will never have to work again. In October 1983, I received a letter in the mail, telling me how to earn $50,000 any time I wanted to. Of course, I gave it a try. Today I am rich. You can be too, if you believe and act on this opportunity.

This is a legitimate business opportunity, a perfectly legal money-making program. If you believe that some day you will et that lucky break, simply follow these instructions and your dream will come true!!! Follow these instructions and in 20-60 days, you will receive $50,000 in cash.

1. Immediately mail $2.00 to each of the five names below. Send cash, check, or money order. Also, send a note to each that says, "Please add my name to your mailing list." This is a service that you are requesting and you are paying $2.00 for this service.

2. Remove the name in the number one spot, move the others up, and place your name and address in the number five position.

3. Copy, print or type 100 more copies of this letter with your name in the number five position.

4. Get a list of 100 names from : National Promotions, Inc., PO box 1547, Bloomfield, NJ 07003 (200 gummed name and address labels for $12.00, 100 for $6.00). Allow 7 days mailing time. Positive thinking works. Do it!

5. While you are waiting for the mailing list to arrive, place the letters in the envelopes and stamp. Do not put your return address on the envelopes.

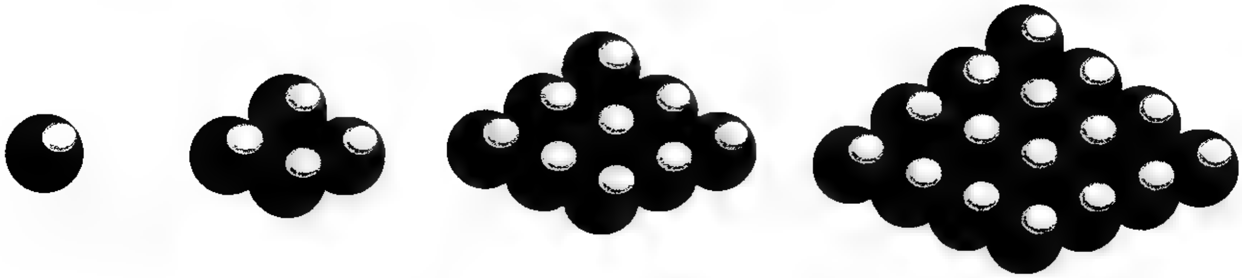
6. When your list arrives, place a label on each of the 100 or more envelopes and drop them in the mailbox.

<u>Within 60 days you will receive over $50,000 in cash!</u>

---!---

1. Al & Paula Goetz, CH-01, Box 235, Kingston, ID 83838
2. Guy V. Davis, 7201 Forestwind Cr., Arlington, TX 76017
3. Lou Earlabaugh, 3701 Seascape Dr., Huntington Beach, CA 92649
4. Mary O'Neill, PO Box 561, Maspeth, NY 11378
5. Lou Galatola, 150 Greenway Terrace, Forest Hills, NY 11375

Cannonball Pyramids*



1 5 cannonballs 14 cannonballs 30 cannonballs

One method of stacking cannonballs is to form a pyramid with a square base. The first four pyramids are shown above.

1. How many cannonballs are there in the fifth pyramid? In the sixth pyramid?

2. How many cannonballs are in the tenth pyramid of cannonballs? Explain how you came up with this.

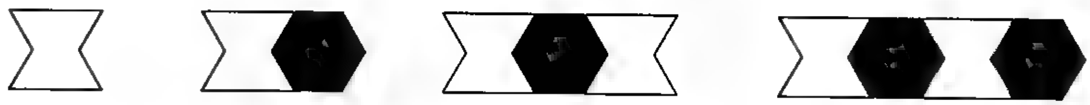
3. Write an expression for the number of cannonballs in the twentieth pyramid. (Note: It is not necessary to compute the actual number.)

4. For any whole number *n*, write an expression for the number of cannonballs in the n^{th} pyramid.

* Adapted from *Mathematics: A Conceptual Approach* by Bennett/Nelson, Wm C. Brown Publishers.

Pattern Block Trains

1. The following are the first four trains in a sequence

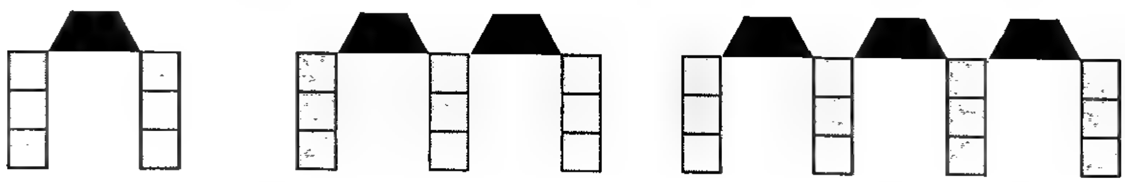


   a) Describe the fifth train in the sequence.

   b) Tell how the first four trains help you to know what the 10th train looks like without having to build all the intervening trains.

   c) Write a description of the twentieth train, so that a person who reads your description could accurately build that train .

2. The first three trains in a sequence are shown below :



   a) Tell how these 3 trains help you to decide what the 20th train looks like without having to build all of the intervening trains.

   b) Describe what you think the 100th train in the sequence looks like so that a person who reads your description could accurately build that train.

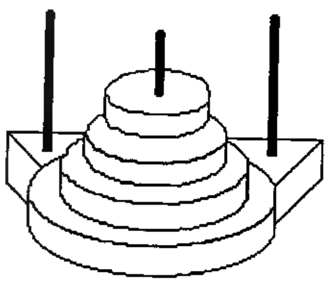
The Tower of Brahma

The Tower of Brahma is an old puzzle from the East. It is based on the legend that at the great temple at Benares in India, there is a brass plate with three diamond pins. On the pins are 64 solid gold discs of decreasing size. At the dawn of creation, the god Brahma put all of the discs on one pin, with the largest disc at the bottom, and the smallest disc on the top. The Hindu priests carry the discs from peg to peg, moving only one disc at a time, and making sure never to place a larger disc on top of a smaller one. According to the legend, when all sixty-four discs have been moved from the first pin to the last pin, the universe will come to an end.

In the toy version, there are still three pins but a smaller number of discs (usually somewhere between four and eight). The task is to move the discs from peg to peg so that they end up stacked in a neat orderly pile on a different peg. Just like the Hindu priests, you can only move one disc at a time, and you can never put a larger disc on top of a smaller one.

Most people new to this puzzle have a lot of difficulty at first. It may help you to start with just two discs. How many moves does it take? Now try three discs. Once you've got it, count the moves. Now try four discs, etc. Make sure that you're finding the fewest number of moves in each case.

1. Make a table showing the fewest moves for one, two, three, four, etc. discs. What patterns do you see?
2. Can you find a recursive formula for the (smallest) number of moves necessary to move n discs?
3. Can you find an explicit formula for the (smallest) number of moves necessary to move n discs?



Counting Cubes

1. You put some unit cubes together to make a 3x3x3 cube.
   a) How many unit cubes do you need?
   b) How many squares show on each face?
   c) How many cubes can you see on the outside?
   d) How many cubes are "hidden" inside?

2. You put some unit cubes together to make a 4x4x4 larger cube.
   a) How many unit cubes do you need?
   b) How many squares show on each face?
   c) How many cubes can you see on the outside?
   d) How many cubes are "hidden" inside?

3. You put some unit cubes together to make a 10x10x10 cube.
   a) How many unit cubes do you need?
   b) How many squares show on each face?
   c) How many cubes can you see on the outside?
   d) How many cubes are "hidden" inside?

4. You put some unit cubes together to make a $n \times n \times n$ cube.
   a) How many unit cubes do you need?
   b) How many squares show on each face?
   c) How many cubes can you see on the outside?
   d) How many cubes are "hidden" inside?

Sums

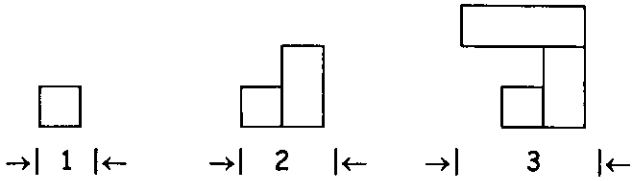
1. Find the sum of the first 1000 counting numbers (1, 2, 3, . . .).

2. If Sarah saves 7 cents the first week of the year, 14 cents the second, 21 cents the third, etc., how much will she have after one year (52 weeks) - remember to add all the amounts together?

3. What is $3 + 6 + 9 + 12 + \ldots + 36000$?

Recursion

1. Construct the next three figures in this recursion.



2. In this problem, the initial values of a sequence will be given together with a rule for determining the values that follow in the sequence. For each, find the first ten terms in the sequence.
   a) *Initial values*: 1 and 3
      Rule: Each of the next terms is the sum of the two preceding terms.

   b) *Initial values*: 2 and -3
      Rule: Each of the next terms is the sum of the two preceding terms.

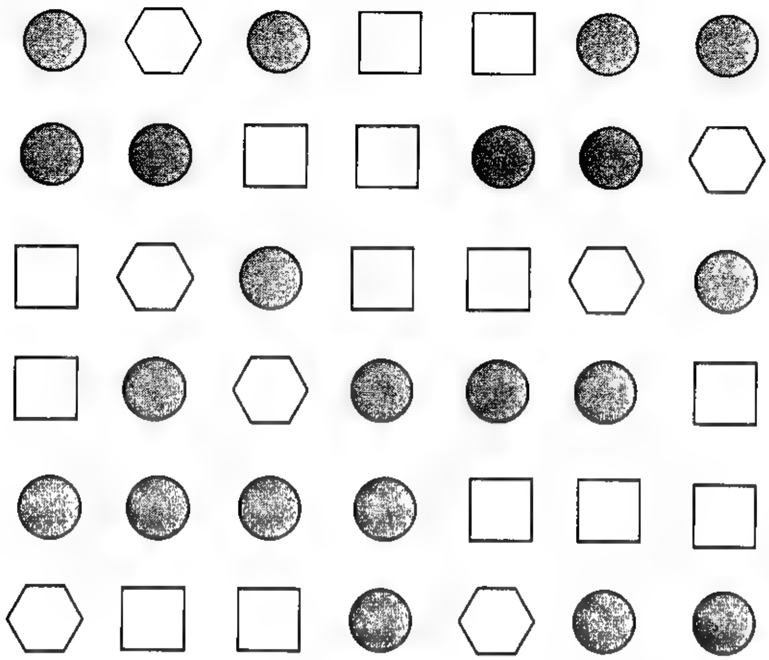
   c) *Initial values*: 1, 2, and 3
      Rule: Each of the next terms is the sum of the three preceding terms.

   d) *Initial values*: 2 and 1
      Rule: Each of the next terms is the product of the two preceding terms.

   e) *Initial values*: 4 and 3
      Rule: Each of the next terms is found using the two preceding terms by multiplying the first term by 2 and adding the next one.

   f) *Initial values*: 4 and 3
      Rule: Each of the next terms is the quotient of the two preceding terms (the first term divided by the second term).

What is the Pattern?*



What number comes next?**

1   11   21   1211   111221   312211

* Created by H. Timberlake.
** N. Rich's favorite sequence!

Rabbits, Bees, and Bricks

Rabbits

A pair of newborn rabbits takes two months to become mature and ready to reproduce. The pair can reproduce a new pair every single month after that. If each new pair of rabbits has the same reproductive habits as its parents, and none of them dies, how many pairs will there be at the end of ten months?

Use solid figures to represent rabbits that are mature at the end of the month (and ready to reproduce), and open figures to represent rabbits that are still immature at the end of the month.

Month	Rabbits	Number of pairs
1		1
2		1
3		2
4		
5		
6		
7		
8		
9		
10		

Do you see a pattern? How many pairs of rabbits will there be at the end of twenty-four months?

Can you figure out why your pattern works?

Bees

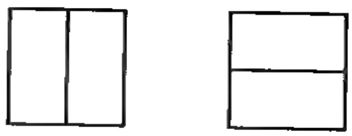
Male bees only have a mother, while female bees have both a mother and a father. How many grandparents does a male bee have? How many great-grandparents? How many great-great-grandparents? Keep going until you know the number of great-great-great-great-great-great grandparents. Do you see a pattern?

Bricks

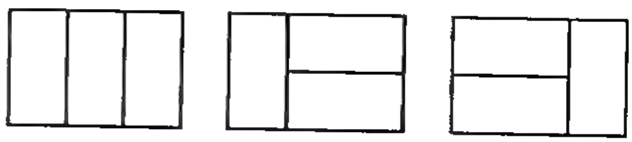
You've become quite experienced in your job as a bricklayer. You decide to amuse yourself by building walls in different patterns. After a while, you start to wonder how many different patterns there are. Assume that your bricks are the standard 4" x 8", and you are building a wall that is 8" high.

If you want the wall to be 4" long, there is just one way to build the wall:

If you want the wall to be 8" long, there are two ways to build it:



If you want the wall to be 12" long, there are three ways to build the wall:



How many different ways can you build the wall if it is 16" long? 20"? 24"?
***BE CAREFUL! This pattern is NOT 1,2,3,4,5,6...

Fibonacci and Pineapples

The sections on the surface of a pineapple spiral around, much like leaves do on a stem. There are two kinds of spirals:

gradual

steep

The number of gradual spirals, the number of steep spirals, and the total number of spirals are almost always (about 99% of the time) consecutive Fibonacci numbers!

Look at some pineapples and check for yourself:

Number of gradual spirals	Number of steep spirals	Total number of spirals

The same is true of pine cones, but it is harder to see the spirals clearly.

To find other Fibonacci numbers in nature:

- count the number of petals on a flower
- slice an apple in half along the "equator" and look at the seeds
- look on a web page (I suggest `http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/`) and find something interesting.

Investigation of Fibonacci Numbers

Write out the ratios 1/1, 2/1, 3/2, 5/3, 8/5, 13/8, 21/13, 34/21, 55/34, 89/55, 144/89, 233/144, 377/233 to ten decimal places (This is easiest to do on a spreadsheet).

What do you notice?

Using a ruler or other measuring device, each person should measure and record the results as follows (the letter *d* stands for *distance*).

1. ______________ ÷ ______________ = ______________
   d (navel to floor) ÷ *d* (top of head to navel)

2. ______________ ÷ ______________ = ______________
   height ÷ *d* (navel to floor)

3. ______________ ÷ ______________ = ______________
   d (right shoulder to end of little finger on right hand) ÷ *d* (left shoulder to right shoulder)

4. ______________ ÷ ______________ = ______________
   d (elbow crease to fingertips) ÷ *d* (shoulder to elbow crease)

Leonardo Fibonacci
c. 1175 - 1250

Patterns in the Fibonacci Sequence

The first ten terms in the Fibonacci sequence are listed below.

1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}	8^{th}	9^{th}	10^{th}
1	1	2	3	5	8	13	21	34	55

1. Copy this list and continue it to the 20^{th} term.

2. Notice that every third term of the Fibonacci sequence is evenly divisible by two. Use your list to figure out which terms are evenly divisible by

   a) three

   b) five

   c) eight

   d) thirteen

   e) Which terms of the sequence do you suppose are evenly divisible by 55?

3. $1 + 1 = 2$

   $1 + 1 + 2 = 4$

   $1 + 1 + 2 + 3 =$ ___

   $1 + 1 + 2 + 3 + 5 =$ ___

   $1 + 1 + 2 + 3 + 5 + 8 =$ ___

   $1 + 1 + 2 + 3 + 5 + 8 + 13 =$ ___

   $1 + 1 + 2 + 3 + 5 + 8 + 13 + 21 =$ ___

   a) Fill in the missing numbers.

   b) Compare the sums with the list from Problem 1. What do you notice?

   c) Use what you noticed to guess the sum of the first 12 terms of the sequence without adding them.

4. $1^2 + 1^2 = 2$
$1^2 + 2^2 = 5$
$2^2 + 3^2 = \_\_\_\_$
$3^2 + 5^2 = \_\_\_\_$
$5^2 + 8^2 = \_\_\_\_$
$8^2 + 13^2 = \_\_\_\_$
$13^2 + 21^2 = \_\_\_\_$

a) Fill in the missing numbers.

b) What do you notice?

c) Write the next line of the pattern.

5. $1^2 + 1^2 = 2 = 1 \cdot 2$
$1^2 + 1^2 + 2^2 = 6 = 2 \cdot 3$
$1^2 + 1^2 + 2^2 + 3^2 = 15 = 3 \cdot 5$
$1^2 + 1^2 + 2^2 + 3^2 + 5^2 = 40 = 5 \cdot 8$

a) Write the next line of the pattern.

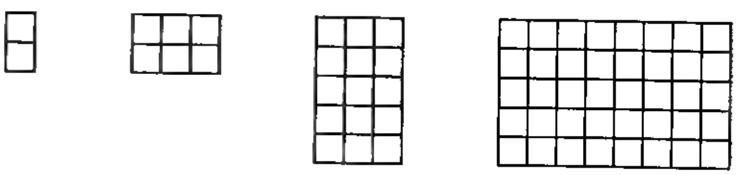
b) Use the pattern to guess the sum of the squares of the first ten terms of the Fibonacci sequence without adding them.

6. $1^3 + 2^3 - 1^3 = 8$
$2^3 + 3^3 - 1^3 = 34$
$3^3 + 5^3 - 2^3 = \_\_\_\_$
$5^3 + 8^3 - 3^3 = \_\_\_\_$

a) Fill in the missing numbers, and write the next line of the pattern

b) What do you notice?

7. Look at the figures below.



a) Do they represent the pattern in Problem 3, 4, 5, or 6?

b) Draw the next figure.

Fun with Fibonacci-like Sequences

1. Select any two numbers (other than 0 and 0) as initial values.
2. Each term in the sequence is the sum of the previous two terms.
3. Produce the first 100 terms on a spreadsheet.
4. Divide the 100th term by the 99th term.
5. Choose two different numbers and do this again. And again. And again.
6. What do you notice?

Bernoulli's Table

This table was published in 1713 in the book *Artis Conjectandi* by Jacob Bernoulli. Do you see any interesting patterns? Can you figure out what the next row will be?

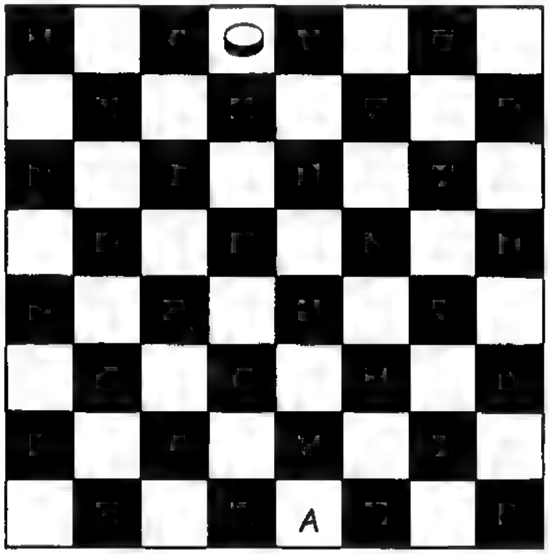
1	1	1	1	1	1	1	1	1	1	1	1
1	2	3	4	5	6	7	8	9	10	11	12
1	3	6	10	15	21	28	36	45	55	66	78
1	4	10	20	35	56	84	120	165	220	286	364
1	5	15	35	70	126	210	330	495	715	1001	1365
1	6	21	56	126	252	462	792	1287	2002	3003	4368
1	7	28	84	210	462	924	1716	3003	5005	8008	12376
1	8	36	120	330	792	1716	3432	6435	11440	19448	31824
1	9	45	165	495	1287	3003	6435	12870	24310	43758	75582
1	10	55	220	715	2002	5005	11440	24310	48620	92378	167960

Jacob (Jacques) Bernoulli
1654 - 1705

Moving Checkers

The checkerboard shown below contains one checker. The checker can only move diagonally "down" the board along the white squares. In how many ways can this checker reach the square marked "A"?

Hint: One way to keep track of this is to write in each white square the number of ways that the checker could move to that particular square. Start at the top, and work your way down towards "A", looking for a pattern.



How Many Ways?

1. Take four different colored blocks. How many ways could you pick two of the blocks? Assume that "one red block, one blue block" is one way, whether you pick the red one first or the blue one first.

2. How many different ways could you pick three blocks out of the four?

3. The symbol $\binom{4}{2}$ means "the number of ways to pick two objects out of four" (it's like a square root symbol). So for Problem 1, you found out what $\binom{4}{2}$ was equal to, and in Problem 2 you found out what $\binom{4}{3}$ was equal to. What is $\binom{5}{2}$ equal to?

4. Finish filling in the answers below. Mathematicians say that $\binom{0}{0}$, $\binom{1}{0}$, $\binom{2}{0}$, etc. are equal to 1, since there is only one way to pick nothing out of a pile (and because it makes the triangle below look nicer ☺)

$$\binom{0}{0}=1$$

$$\binom{1}{0}=1 \qquad \binom{1}{1}=\_\_\_$$

$$\binom{2}{0}=1 \qquad \binom{2}{1}=\_\_\_ \qquad \binom{2}{2}=\_\_\_$$

$$\binom{3}{0}=1 \qquad \binom{3}{1}=\_\_\_ \qquad \binom{3}{2}=\_\_\_ \qquad \binom{3}{3}=\_\_\_$$

$$\binom{4}{0}=1 \qquad \binom{4}{1}=\_\_\_ \qquad \binom{4}{2}=\_\_\_ \qquad \binom{4}{3}=\_\_\_ \qquad \binom{4}{4}=\_\_\_$$

5. Does this triangle look familiar? Use the pattern to guess how many ways you could pick three objects out of six.

Patterns in Pascal's Triangle

1. Draw rows 0 to 7 of Pascal's Triangle.

2. Find the values of 11^2, 11^3, and 11^4. What is the relationship between these numbers and Pascal's Triangle? Will 11^5 fit the pattern? Explain.

3. Look at Diagonal 1. What is the sum of
   its first two numbers?
   its first three numbers?
   its first four numbers?
   its first five numbers?
   its first six numbers?
   What do you observe about these results?

4. Look at Diagonal 2. What is the sum of
   its first and second numbers?
   its second and third numbers?
   its third and fourth numbers?
   its fourth and fifth numbers?
   its fifth and sixth numbers?
   What do you observe about these results?

5. What are the first three numbers in the 18^{th} row of Pascal's Triangle?

6. This pattern is called the harmonic triangle. In what ways is the harmonic triangle related to Pascal's Triangle?

$$1$$
$$\frac{1}{2} \quad \frac{1}{2}$$
$$\frac{1}{3} \quad \frac{1}{6} \quad \frac{1}{3}$$
$$\frac{1}{4} \quad \frac{1}{12} \quad \frac{1}{12} \quad \frac{1}{4}$$
$$\frac{1}{5} \quad \frac{1}{20} \quad \frac{1}{30} \quad \frac{1}{20} \quad \frac{1}{5}$$
$$\frac{1}{6} \quad \frac{1}{30} \quad \frac{1}{60} \quad \frac{1}{60} \quad \frac{1}{30} \quad \frac{1}{6}$$

7. The sums of certain diagonals in Pascal's triangle will be the Fibonacci numbers. Can you find out which diagonals? (Hint: This may be easier if you write Pascal's triangle as a right triangle).

Blaise Pascal
1623 - 1662

Alphabet Soup*

1. You decide to put all of the letters of the alphabet into piles. Two or more letters will go in the same pile if and only if one of the letters can be reshaped into the other (and vice versa) by "stretching" or "bending" the letter, but without "gluing" or "tearing" it. How many different piles are there?

A B C D E F G H I J K L M
N O P Q R S T U V W X Y Z

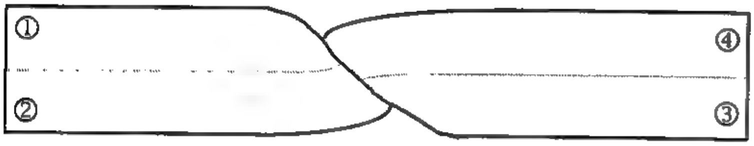
2. This time, two or more letters will go into the same pile if and only if they enclose the same number of chambers. How many different piles are there now?

* From the Math 130 course packet at the University of Wisconsin at Madison.

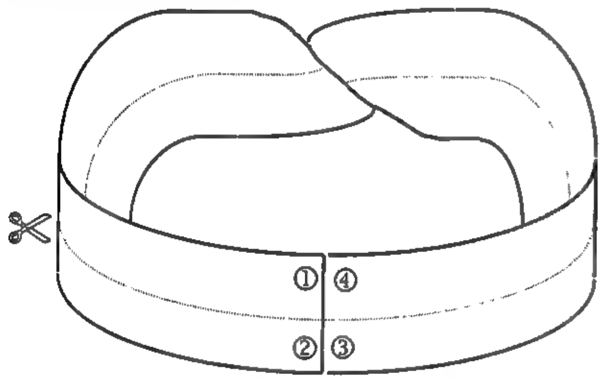
Möbius Strips

A Möbius strip is a ring of paper with a half-twist in it. Start with a strip of paper and draw a dotted line down the middle on both sides.

Twist the paper once.



Now join the ends of the paper, like you were making a paper chain. Tape the ends together on both sides (be sure to use enough tape to cover the whole seam).



Trace on the dotted line with a marker: how many sides does a Möbius strip have? Trace along the outer edge: how many edges does a Möbius strip have? Now cut the Möbius strip in half lengthwise, using the dotted line as a guide. What does it look like? What happens if you cut the Möbius strip in half again?

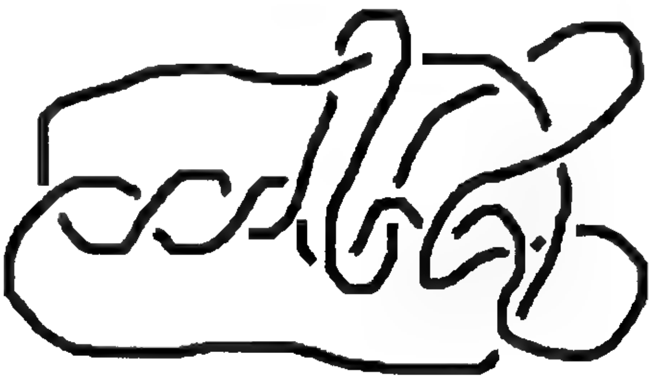
Augustus Ferdinand Möbius
1790 - 1868

A Knotty Problem*

This is a knot:

This is an "unknot" (since you can untwist it into a loop):

Below is a knot with fourteen crossings. If you change two of the crossings, it becomes an "unknot". You change the crossings by altering which part of the loop lies on top at the crossings. Which two crossings do you need to change?

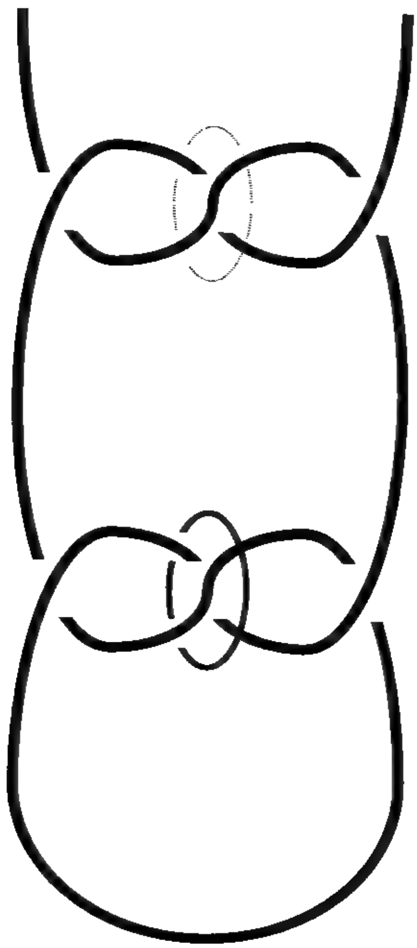


* From page 63 of *Puzzles, Mazes, and Numbers* by Charles Snape and Heather Scott, New York: Cambridge University Press, 1995.

Moving the Ring*

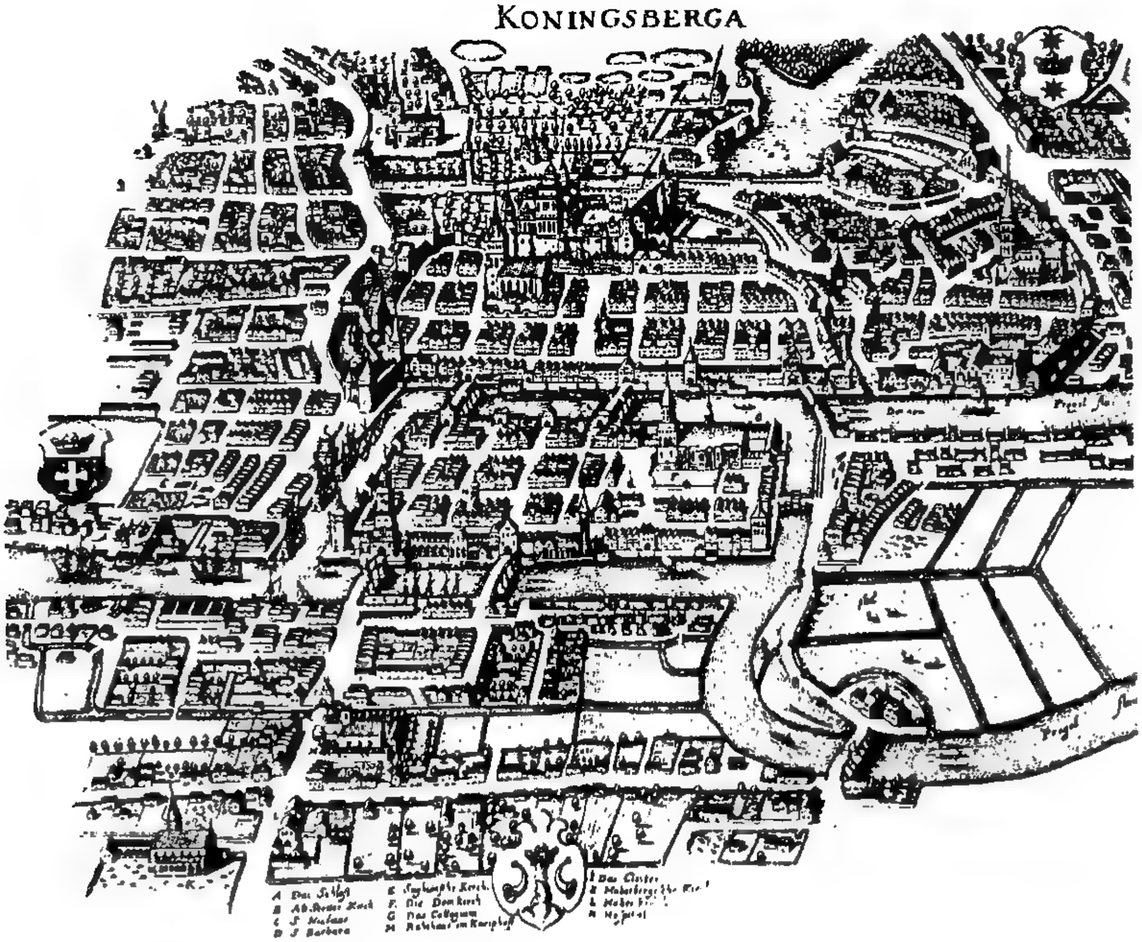
This puzzle was invented by Majunath M. Hegde while he was a student in India.

In the picture below, a rope is knotted and a ring is placed in the middle of the bottom knot. Assume that the top of the rope is tied to something (like furniture) so that the ring can't come off. Can you move the ring and knot around so that the ring ends up in the top position, where the dotted outline is?

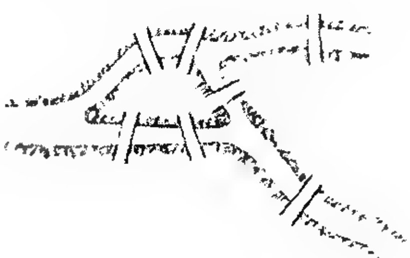


* From page 75 of *The last Recreations* by Martin Gardner, New York: Springer-Verlag, 1997.

The Königsberg Bridge Problem

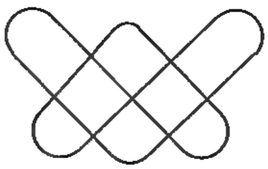


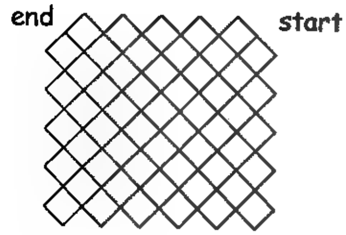
The old city of Königsberg (now Kaliningrad) in East Prussia was located on the banks and two islands of the Pregel River. Seven bridges crossed the river approximately as shown in the diagram. On Sundays the citizens would walk around the town, as is common in German cities. The citizens wondered if they could find a path in which they crossed each bridge exactly once. Is it possible? If it is, is it also possible to find a path that crosses each bridge exactly once and takes you back to where you started?



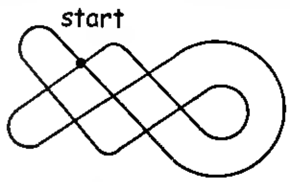
Euler Paths in Many Cultures*

These figures are sand drawings from the Bushoong in Zaire in western Africa. Children trace them in the sand, going along each edge exactly once. Can you find a way to do this? (This problem stumped a European ethnologist who was studying the Bushoong in 1905.)

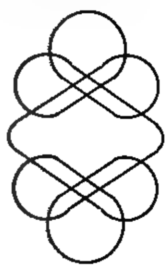




Similar sand tracings occur in the rituals of the Tshokwe, who live in the Zaire/Angola region of western Africa, and are drawn exclusively by men. The figure below was traditionally traced by an elder during the *mukanda*, a rite of passage of boys into adulthood. Can you trace it, going over each line exactly once?

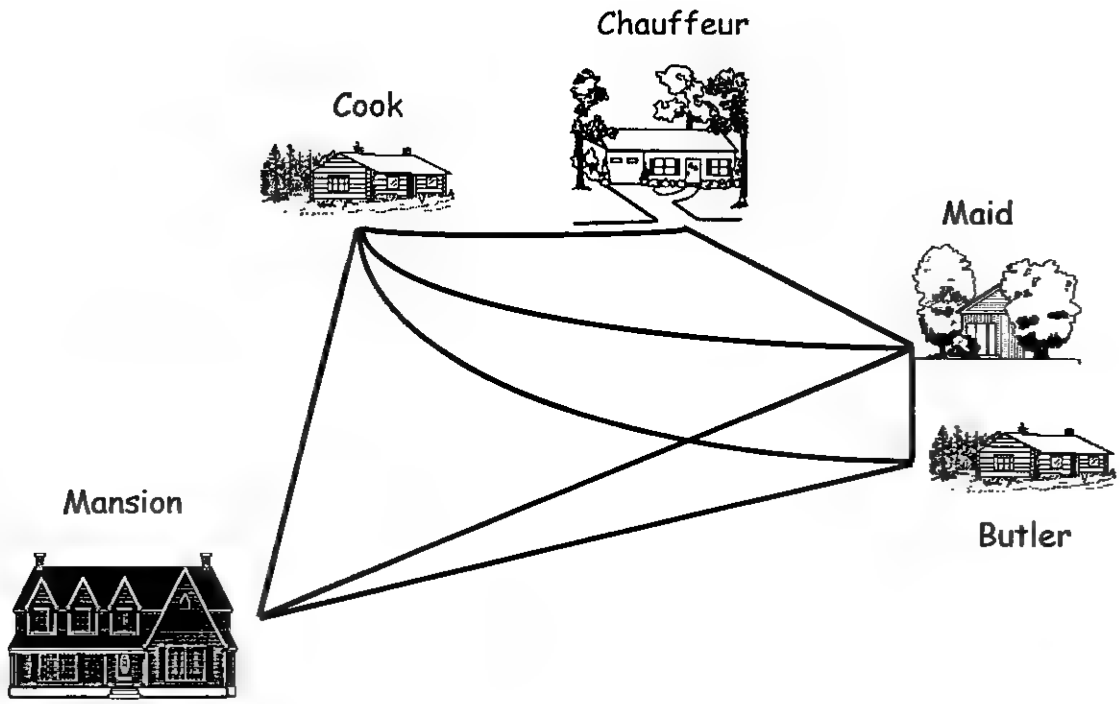


The figure below is from the indigenous people of Malekula, an island in the Republic of Vanuatu between Australia and New Zealand. According to their mythology, when people die they must know how to trace this figure (going over each edge exactly once) or they will not be allowed passage to the Land of the Dead.



* From Chapter Two of *Ethnomathematics* by Marcia Ascher, New York: Chapman & Hall, 1991.

The Case of the Stolen Diamonds*



It was a snowy night when the family jewels were stolen from the Shmendrick Mansion. As soon as the alarm was sounded the gates around the estate were locked. The jewels were somewhere on the grounds. But where? Were they in the cook's quarters, or perhaps in the chauffeur's cottage? There was only one person who could crack this case. World famous inspector Euler Toots of Scotland Yard. Early the next morning Inspector Toots came to visit the estate in search of clues.

This is what he found: fresh tracks crisscrossing the grounds. He couldn't tell which direction they were going, but he was sure that each track had only been used once. He was careful to make a map of the tracks before the snow melted.

At noon, all of the principles in the case were brought into the main dining hall, just as Inspector Toots was hanging the map on the wall. Toots began his questioning.

* This problem has been passed around a bit but the original source is unknown.

Toots: Lady Shmendrick, when did you last see the diamonds?

Lady S: Around midnight, just before I went to bed. I remember looking out of my window. It was just beginning to snow. All of the lights were out in the servants' quarters, so I locked the diamonds up in the safe and went to bed. The next thing I remember is waking up around 4am to the sound of alarms, and discovering my jewels were gone.

Toots: Thank you. It is clear from your testimony that since the lights were out before the snow started, the tracks were probably made by the thief. . . . Mr. Hanson, you're the chauffeur, what do you remember of last night's activities?

Chauffeur: I was sleeping soundly when I was awoken by the alarm. I was still in bed when I heard someone run into my house, then out the other door. I didn't get a chance to see who it was, though. Sorry about that.

Toots: That's O.K. Mr. Hanson. Your testimony seems to fit the evidence on the map (see map). There are two snow tracks connected to your cottage. The thief must have taken one of them going into your cottage and the other while running away.

Cook: I'd like to say something Inspector.

Toots: Yes, Mr. Cromwell?

Cook: I couldn't sleep at all last night. I was thinking about a new dessert recipe that I've been working on. Chocolate Chip Mint Marshmallow Swirl is what I call it. Well, all of a sudden someone ran through my house and right out the other door. Then about five minutes later it happened again. I couldn't tell who it was but I did see that the person was carrying the family jewels.

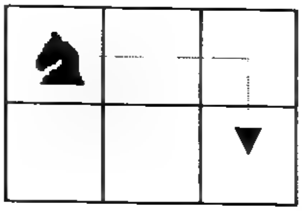
Butler: Why, that's incredible! The same thing happened to me. In and out the person ran, then in and out again.

Maid: The person ran through my house too. I'm not sure how many times, but I know more than once.

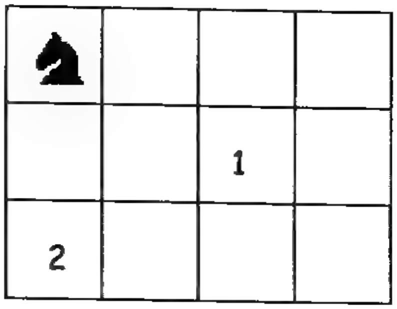
Toots: One of you is a liar and I know who it is!

The Knight's Tour

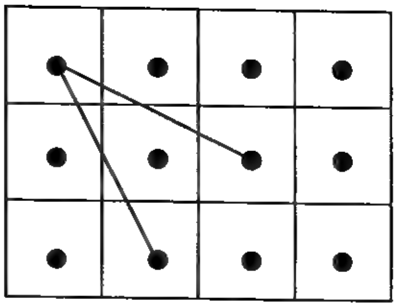
In the game of chess, the knight ("horse") moves in an L-shape (1 square by 2 squares, or 2 squares by 1 square). For example, a legal move is



For many years, people wondered if the knight could move around the chessboard, landing on each square exactly once, using only legal moves. This is called the Knight's Tour problem.*



1. Find a way for this knight to hit every square in this "small chessboard" exactly once. Mark the squares 1, 2, 3, etc. to indicate the order in which the knight jumps on each square (that's easier to follow than drawing arrows). The first two jumps are drawn in.



2. Start again with a new board. You can make a graph by first drawing a vertex for each square on your "small chessboard", and then drawing an edge between vertices if the knight can move between those squares in a legal move. Finish drawing the graph:

3. Use the graph from Problem 2 to show the route that your knight took in Problem 1. What kind of a path is this?

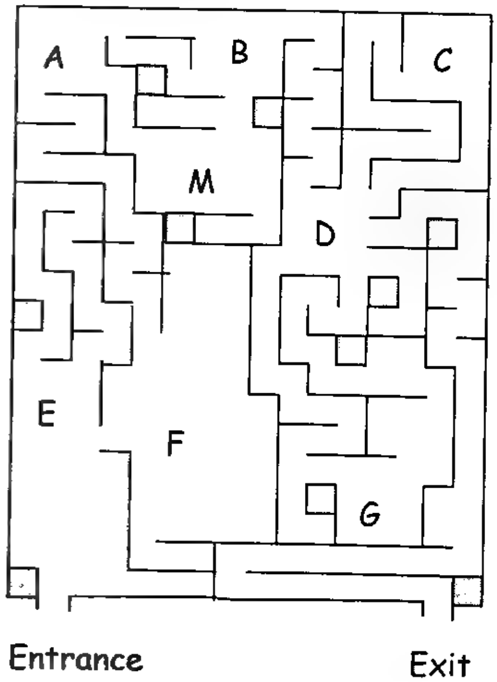
4. Draw a graph for a 4 x 5 chessboard. Can you find a Knight's Tour?

---

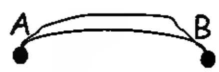
* It can! See the page `http://mathworld.wolfram.com/KnightsTour.html` for some examples.

The Fun House Maze*

An amusement park has a fun house in the form of a maze with the floor plan shown below. The maze contains a room with trick mirrors, labeled M in the floor plan, together with seven smaller rooms.



1. To show how the eight rooms of the maze are connected, draw a network in which the entrance, exit, and the rooms are represented as vertices and the possible paths between rooms are represented as arcs. For example, there are two different path connecting rooms A and B, and so two arcs would be drawn between vertices A and B of your network as shown here.



Use your network to answer the following questions.

2. Can you enter the maze and go through each corridor exactly once? Is this related to Euler paths or Hamilton paths? Explain.

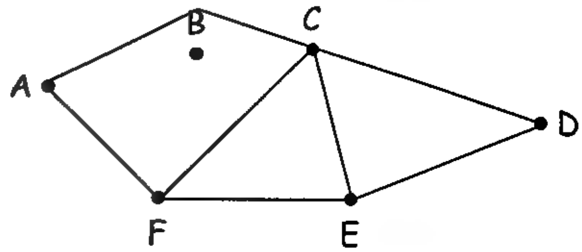
3. Can you enter the maze and go through each room exactly once? Is this related to Euler paths or Hamilton paths? Explain.

---

* This problem has been passed around a bit, but the original source is unknown.

The Traveling Salesman

Rasheed is a salesman, and his is planning his itinerary to visit six companies: A through F. These companies, and the roads between them, can be represented by a graph.

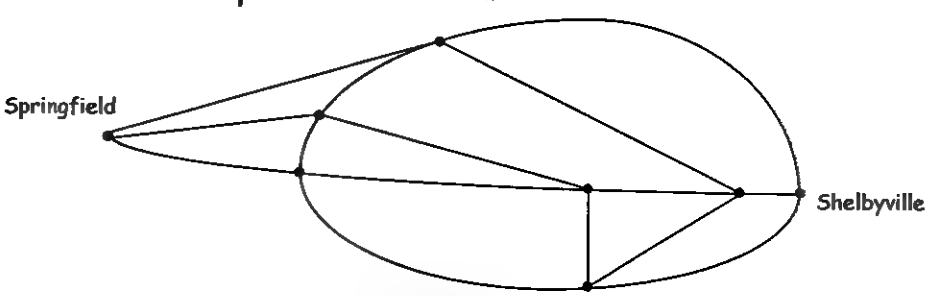


1. Can Rasheed start at once of the companies (A through F) and visit the other companies exactly once? Can Rasheed start at one company and travel along each road exactly once? Which question is about Euler paths and which is about Hamiltonian paths? Explain.

2. Add a seventh Company (G). Mark roads to and from Company G so that Rasheed could start at Company G and travel along each road exactly once.

Highways

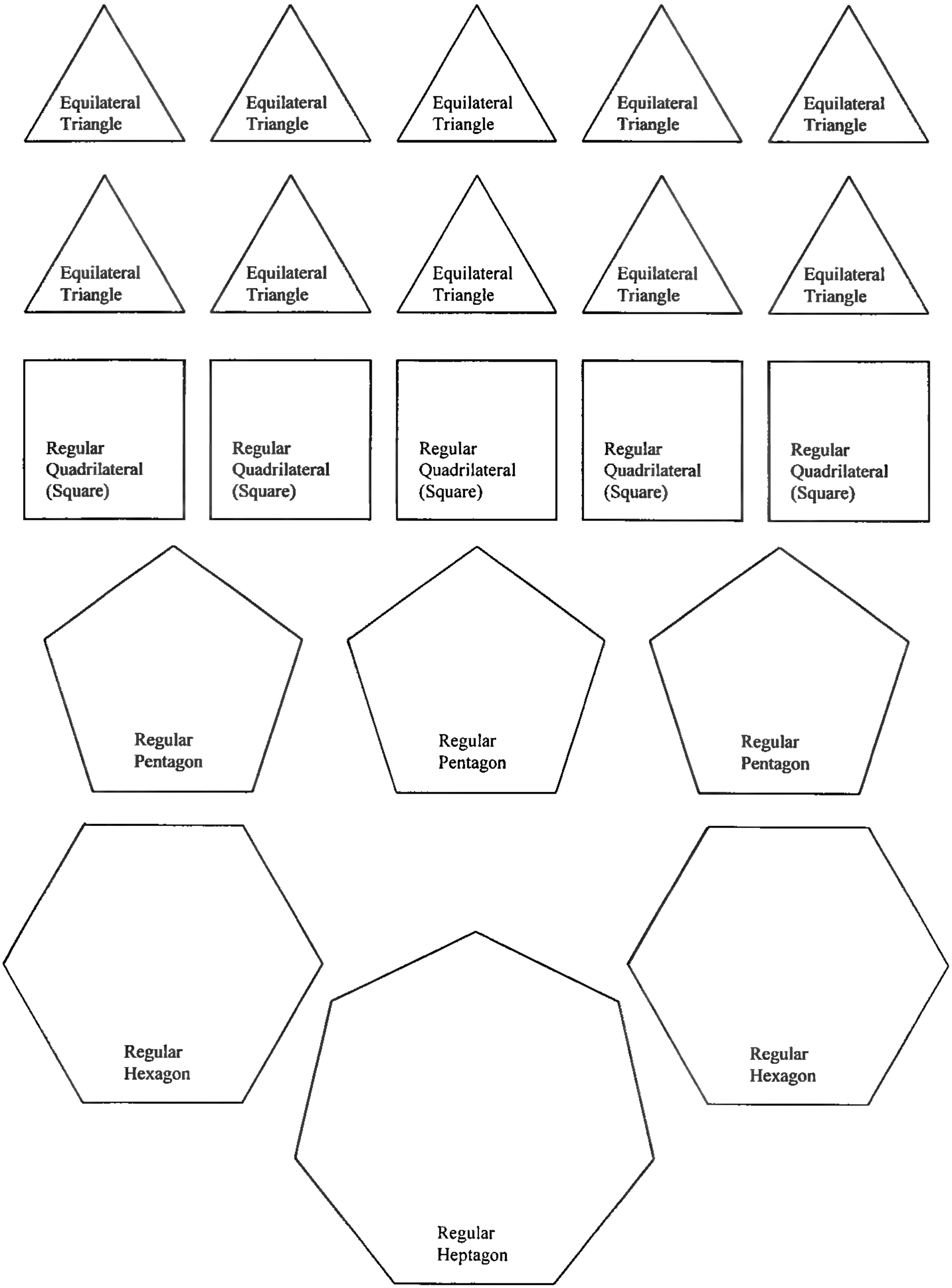
A highway inspector must drive over all the highways in her area to inspect for potholes. The map shows her area.

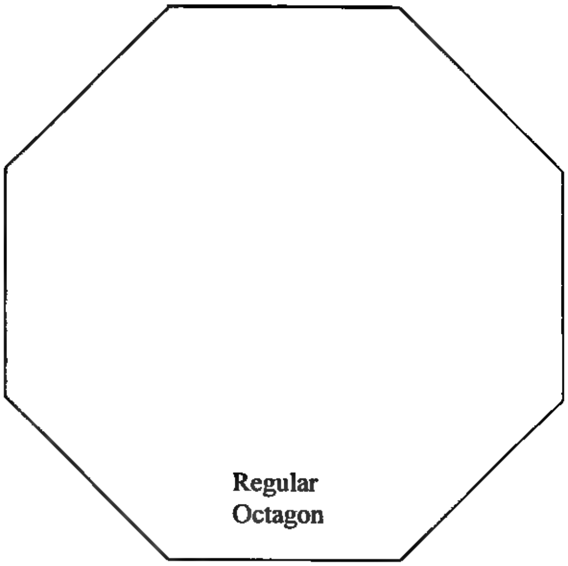


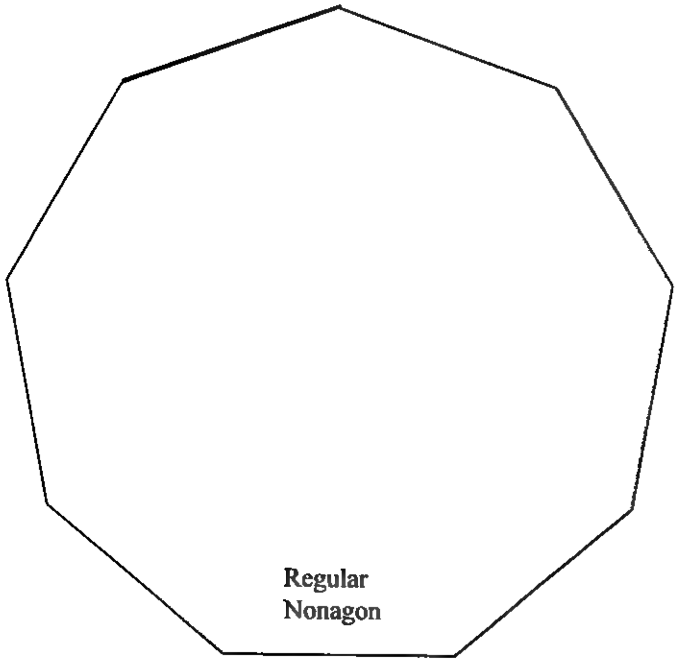
1. Can the inspector leave Springfield, cover each of the highways exactly once, and finish at Springfield?

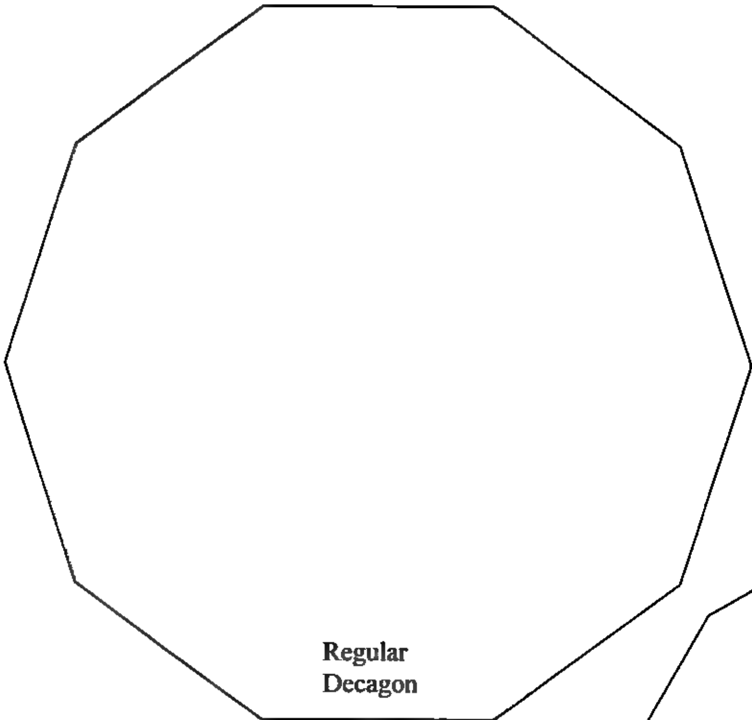
2. Can she leave Springfield, cover each highway exactly once, and finish at Shelbyville?

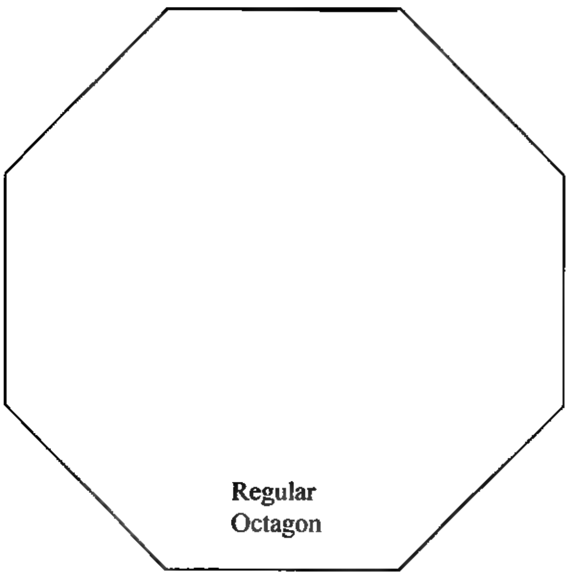
3. Can she leave Springfield and visit each intersection exactly once?

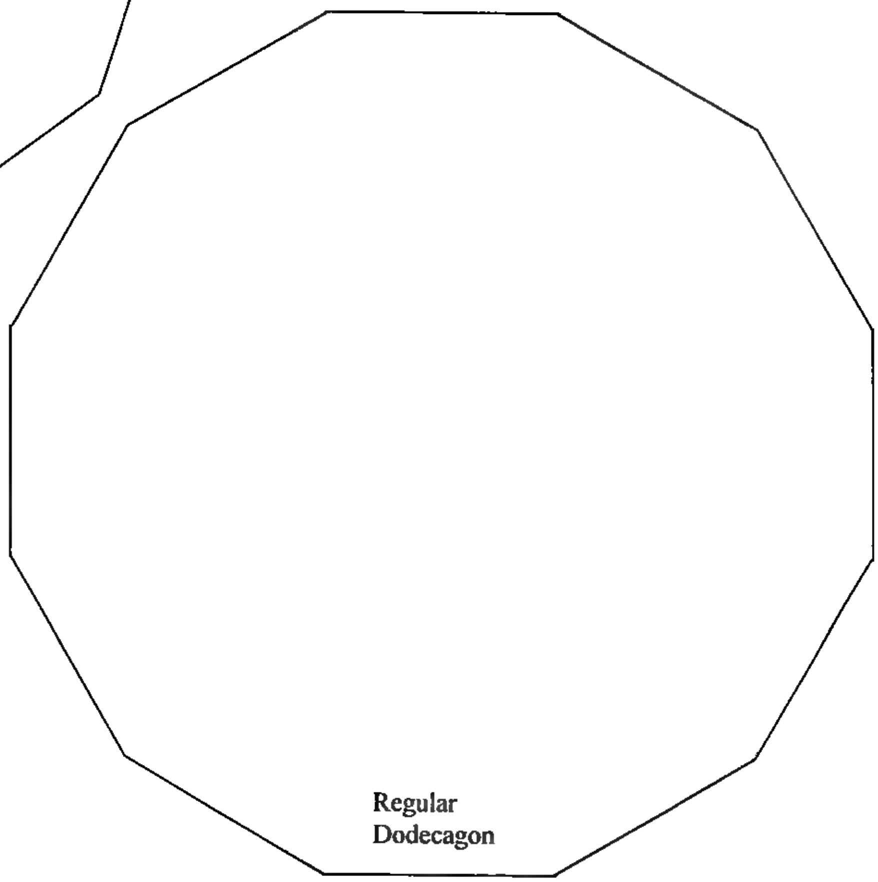

Equilateral Triangle
Equilateral Triangle
Equilateral Triangle
Equilateral Triangle
Equilateral Triangle
Equilateral Triangle
Equilateral Triangle
Equilateral Triangle
Equilateral Triangle
Equilateral Triangle
Regular Quadrilateral (Square)
Regular Quadrilateral (Square)
Regular Quadrilateral (Square)
Regular Quadrilateral (Square)
Regular Quadrilateral (Square)
Regular Pentagon
Regular Pentagon
Regular Pentagon
Regular Hexagon
Regular Heptagon
Regular Hexagon


Regular
Octagon


Regular
Nonagon


Regular
Decagon


Regular
Octagon


Regular
Dodecagon

Use your polygons to discover whether a point can be surrounded by the following combinations of shapes. If it is possible to surround the point, indicate how many of each polygons are used. If it is not possible, say so.

10. Triangles and squares

11. Triangles and hexagons

12. Squares and hexagons

13. Triangles, squares, and hexagons

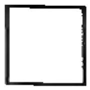
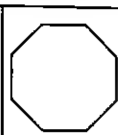
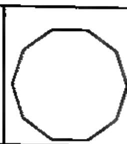
14. Triangles, squares, and octagons

15. A dodecagon and polygons of two other shapes (two different combinations are possible in the case - try to find both of them)

Now try to list all of the ways to surround a point with combinations of two or more regular polygons.

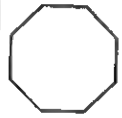
Surrounding a Point with Regular Polygons

Fill in the angle measure of each regular polygon. Afterwards, list all of the ways regular polygons can surround a point by inserting the number of each figure needed. You can use the same type or different types (as in the first example).

Figure								Numerical Designation
Angle Measure								
	3	2						3-3-3-4-4 or 3-3-4-3-4

Surrounding Space with Regular Polygons

Fill in the angle measure of each regular polygon. Afterwards, list all of the ways regular polygons can surround a point in space (not necessarily lying flat in the plane) by inserting the number of each figure needed. Start by only using one type of piece. If you find all of those, try to find the combinations that use two or more different types of pieces.

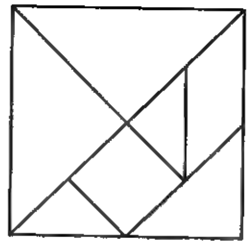
Figure							
Angle Measure							
	3						

Recording Info on Platonic (or Archimedian) Solids

			In the whole figure		
Shape of each face	Number of edges on each face	Number of faces around each vertex	Total number of faces	Total number of edges	Total number of vertices

- What patterns do you notice in the chart? There are a lot of interesting things!
- Is there a mathematical way to figure out the number of edges (or vertices)? You could use this to double check if you aren't sure you counted correctly.

Tangrams



A tangram is a Chinese puzzle made from seven shapes cut out of a square. The pieces can be put together to form other shapes.

1. Can you make a triangle with two tangram pieces? With three pieces? Four pieces? Five pieces? Six pieces? With all seven pieces? Draw and label pictures to show how you made the triangles.

2. How are these triangles different? How are they alike? Do you know special names for any of the triangles?

3. Can you make a quadrilateral with two tangram pieces? Three pieces? Four? Five? Six? All seven pieces? Draw and label pictures to show how you made each quadrilateral.

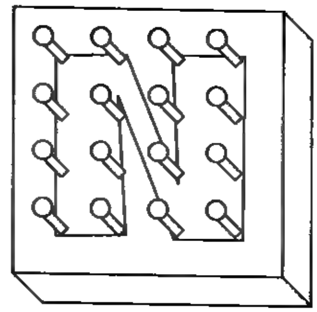
4. How are the quadrilaterals different? How are they the same? Do you know special names for any of the quadrilaterals?

5. Can you make a figure of a person using all seven pieces?

6. Which numbers can you make with all seven pieces?

7. Which letters can you make with all seven pieces?

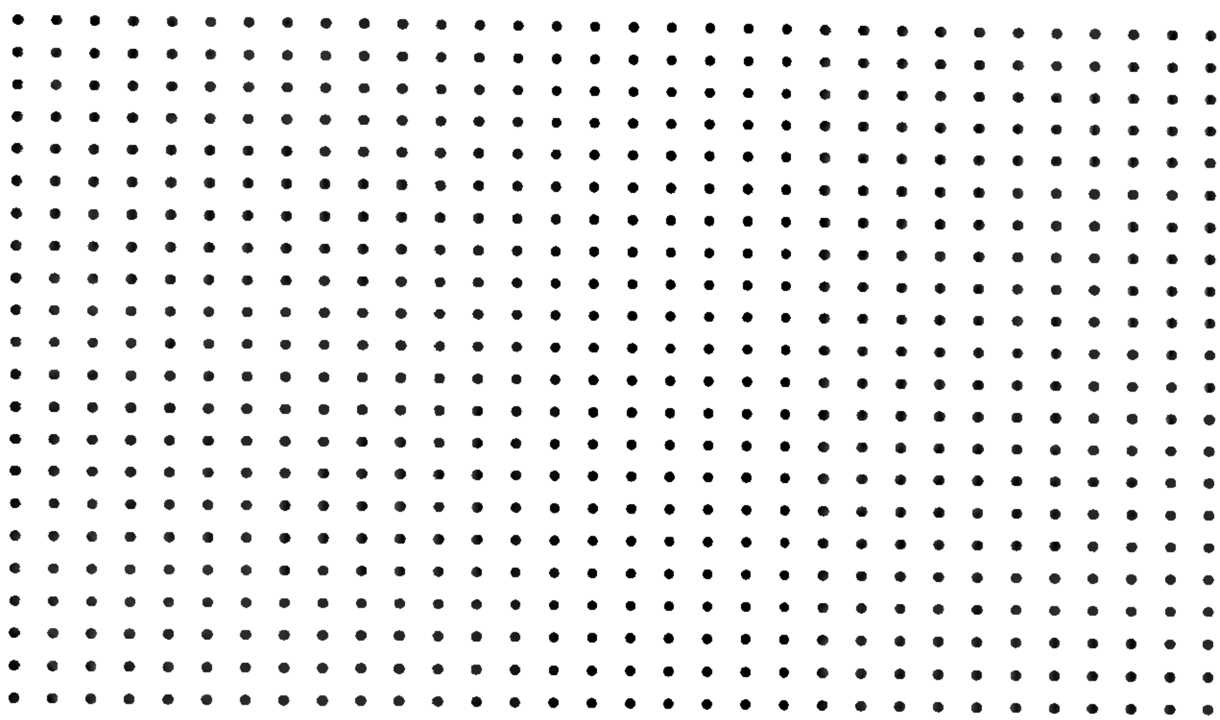
Geoboards*



This figure shows a ten-sided figure on a 4 x 4 geoboard. Note that no side crosses another side and no two vertices coincide.

1. A student searched for such a figure with the greatest number of sides. On a 4 x 4 geoboard she found a sixteen-sided figure. On a 6 x 6 geoboard she found a 36-sided figure. Show these figures on dot paper (or below)

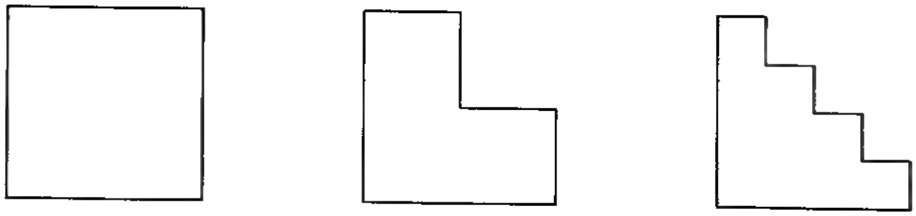
2. Make a conjecture that might be true about the greatest number of sides for such figures on a 2 x 2 geoboard, on a 3 x 3 geoboard, on a 5 x 5 geoboard, and on a 7 x 7 geoboard. Test your conjectures by showing figures with the largest possible number of sides on dot paper (or below). What other conjectures can you make?



* Investigation 3.1 on p. 56 of *Geometry an Investigative Approach, 2nd Ed.*, by Phares G. O'Daffer and Stanley R. Clemens, Addison-Wesley Publishing Company, New York, 1992.

Fractals

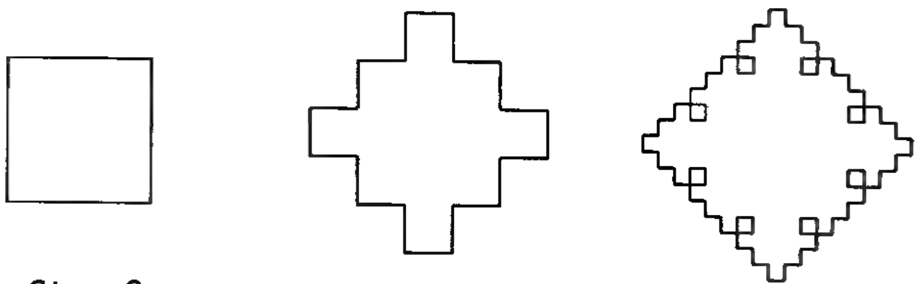
1.



Stage 0 Stage 1 Stage 2

How is each Stage drawn? Draw Stages 3 and 4 of this construction.

2.



Stage 0 Stage 1 Stage 2

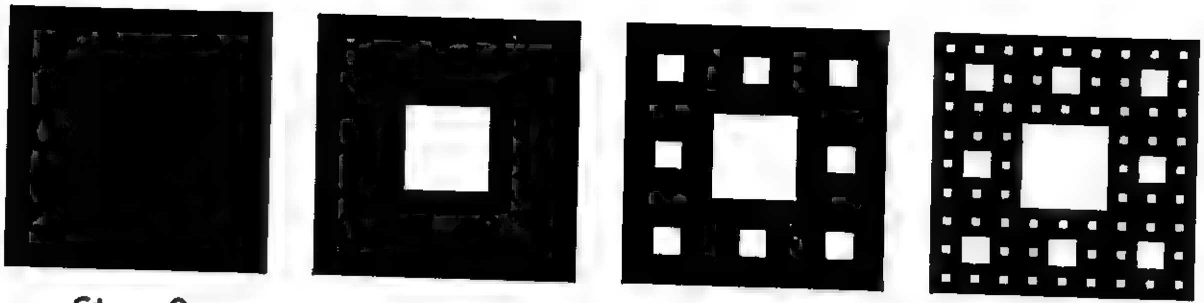
How is each Stage drawn? Draw Stage 3 of this construction.

3. Look at the fractal drawing of a fern below:

What would the first four stages of this drawing look like?

4. Make up your own fractal image. Draw Stage 0 through Stage 3, and describe how each stage is drawn.

The Sierpinski Carpet



Stage 0 Stage 1 Stage 2 Stage 3

1. Use the pattern to predict how many white squares there will be in Stage 4 of the Sierpinski Carpet.

2. Draw Stage 4 of the Sierpinski Carpet. Were your predictions for Problem 1 correct?

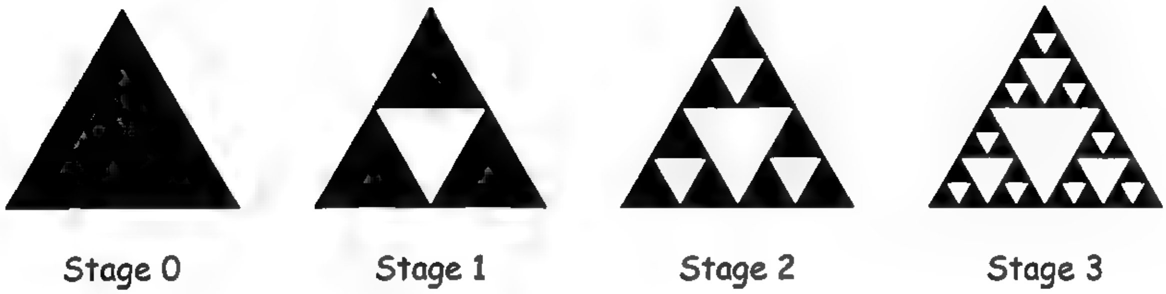
3. How many white squares will there be in Stage 5 of the Sierpinski Carpet?

4. Can you find a formula (or description) for the number of white squares in Stage *n* of the Sierpinski Carpet?

5. With your group, come up with an interesting variation of the Sierpinski Carpet. Answer questions 1 through 4 (modified, if necessary) for your variations.

Waclaw Sierpinski
1882 - 1969

The Sierpinski Triangle



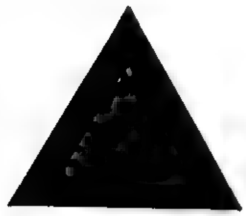
1. Use the pattern to predict how many black triangles and how many white triangles there will be in Stage 4 of the Sierpinski Triangle.

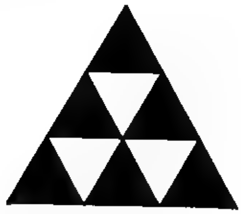
2. Draw Stage 4 of the Sierpinski Triangle. Were your predictions for Problem 1 correct?

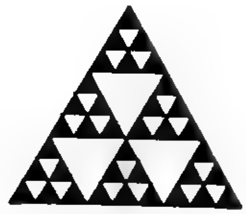
3. How many black triangles will there be in Stage 5 of the Sierpinski Triangle? How many white triangles will there be?

4. Can you find formulas (or descriptions) for the number of black triangles and the number of white triangles in Stage *n* of the Sierpinski Triangle?

A Variation of the Sierpinski Triangle







Stage 0 Stage 1 Stage 2

1. Use the pattern to predict how many black triangles and how many white triangles there will be in Stage 3 of this variation.

2. Draw Stage 3 of the variation. Were your predictions for Problem 1 correct?

3. How many black triangles will there be in Stage 4 of this variation? How many white triangles will there be?

4. Can you find formulas (or descriptions) for the number of black triangles and the number of white triangles in Stage *n* of this variation.

5. With your group, come up with two interesting variations of the Sierpinski Triangle. Answer questions 1 through 4 (modified, if necessary) for one of your variations.

The Chaos Game

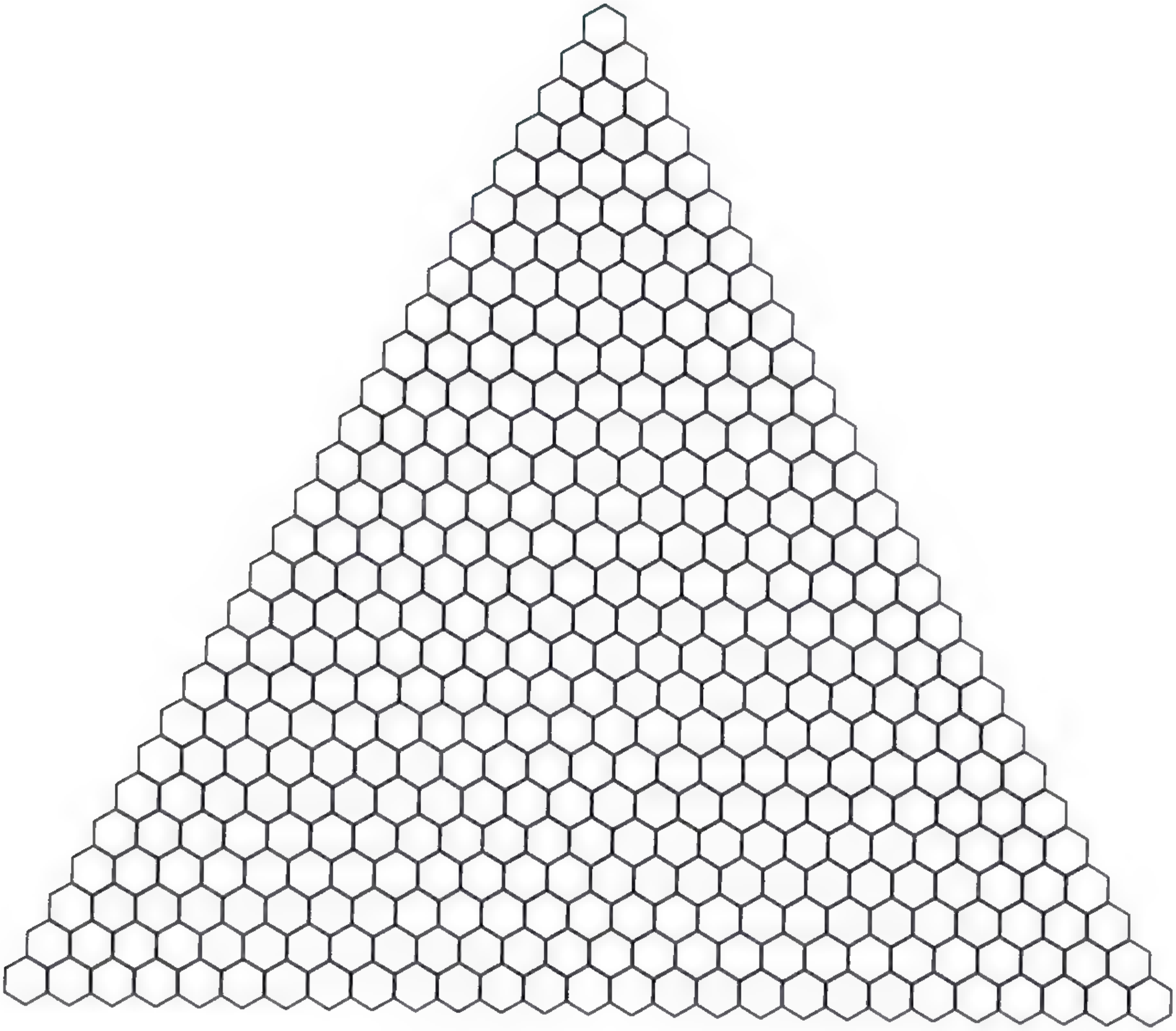
Draw a dot.
Roll the dice.

  If you get a 1 or 2, put a new dot halfway between your previous dot and dot A.
  If you get a 3 or 4, put a new dot halfway between your previous dot and dot B.
  If you get a 5 or 6, put a new dot halfway between your previous dot and dot C.

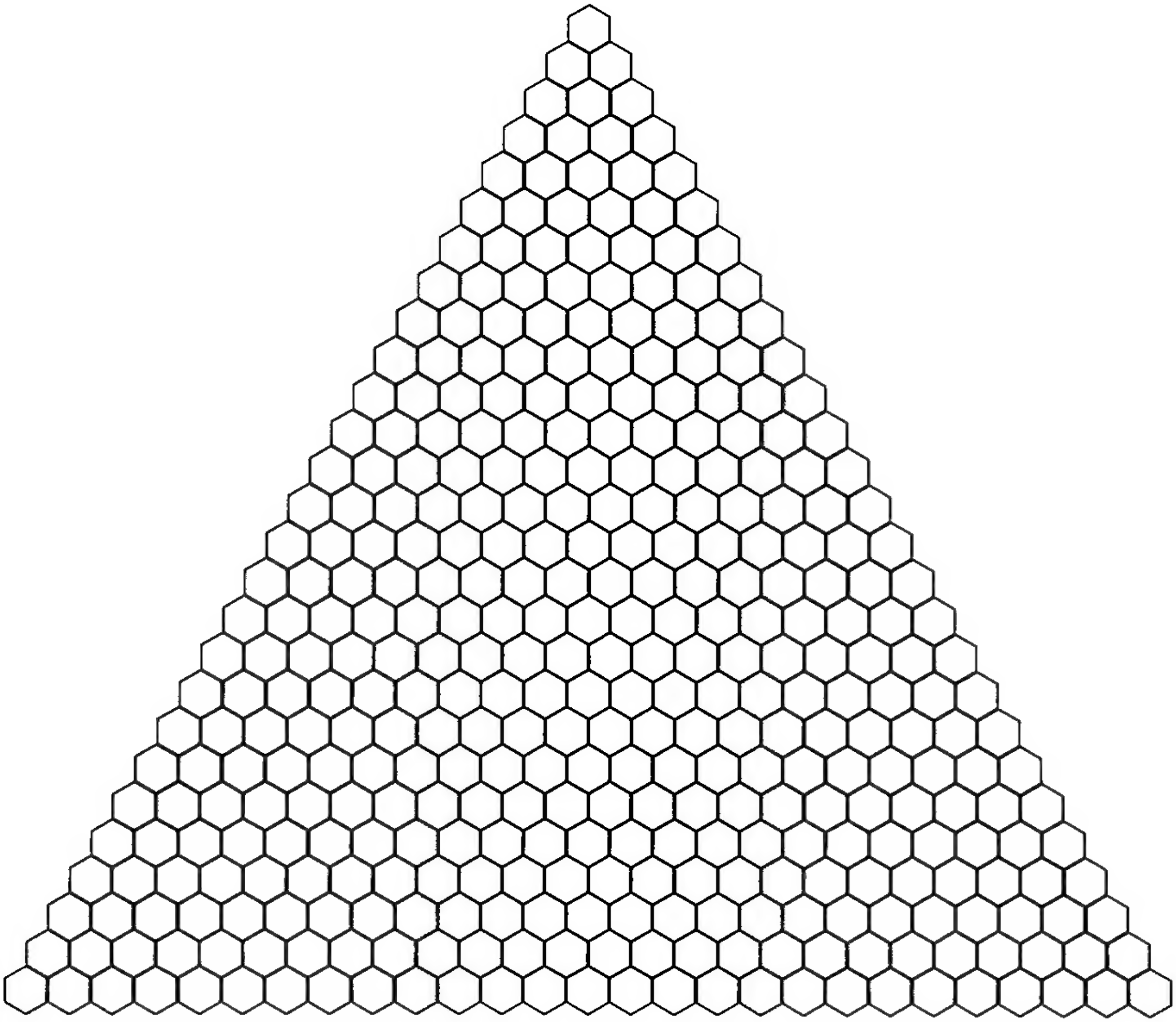
Now you have a new dot!
Roll the dice.

  If you get a 1 or 2, put a new dot halfway between your previous dot and dot A.
  If you get a 3 or 4, put a new dot halfway between your previous dot and dot B.
  If you get a 5 or 6, put a new dot halfway between your previous dot and dot C.

Keep going! After you've drawn 100 or 200 dots, what does the figure look like?

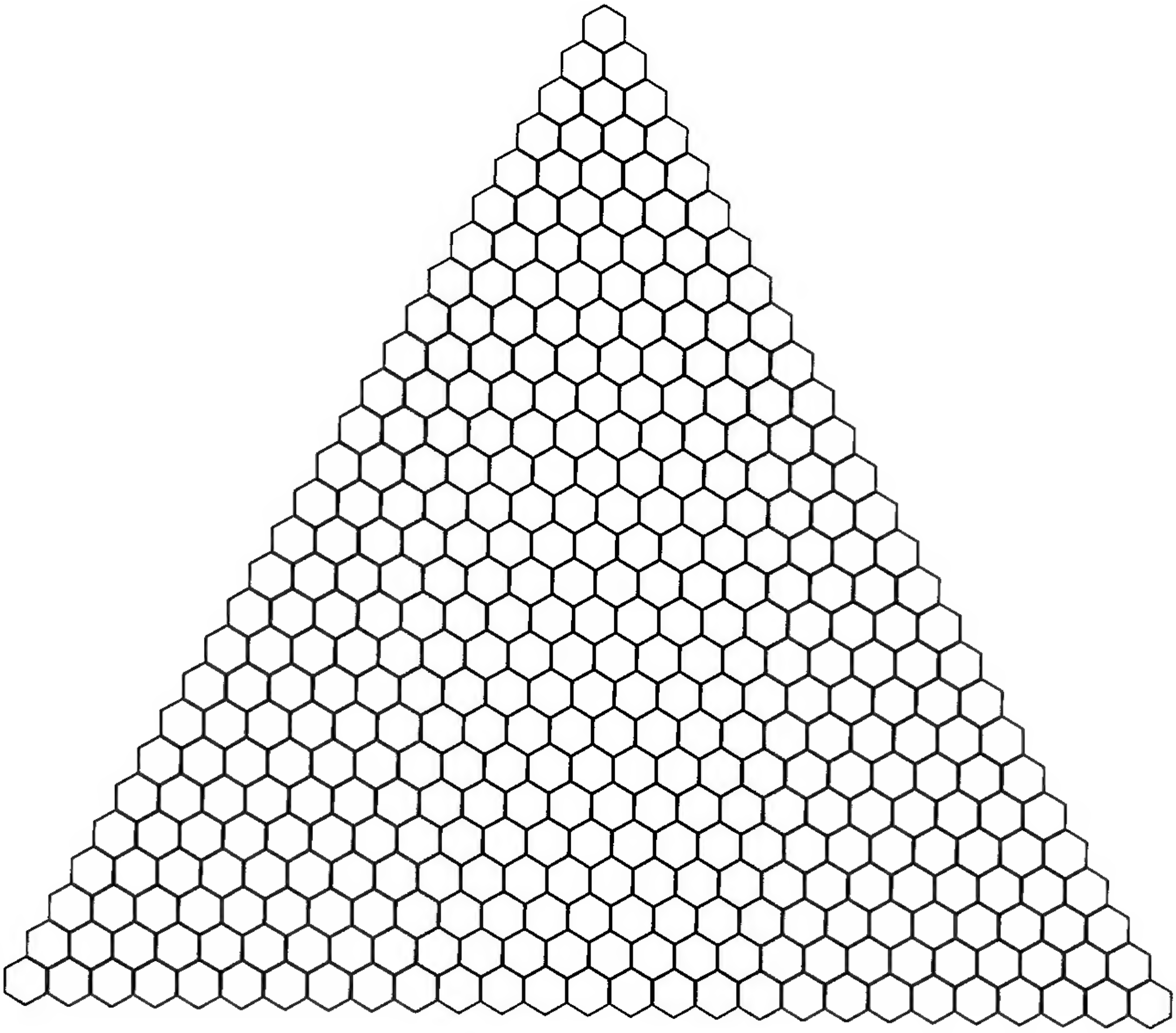


Start by filling in the numbers for Pascal's Triangle. If the number is odd, color that cell black. If the number is even, color that cell white. After a while you may notice that if the two cells next to each other are **different** in color, then the cell below will be shaded black, and if two cells next to each other are the **same** in color, the cell below will be left unshaded. End cells in each row are always colored black.



Refer to the numerical entries of Pascal's Triangle. Express each number in modulo 3 form (write the first numbers using Pascal's Triangle, but then you can save yourself time by using addition modulo 3) and then color in the corresponding cell using the following rule:

If the entry is a 1 or 2, shade the cell **black**.
If the entry is a 0, leave the cell unshaded as **white**.



Refer to the numerical entries of Pascal's Triangle. Express each number in modulo 5 form (write the first numbers using Pascal's Triangle, but then you can save yourself time by using addition modulo 5) and then color in the corresponding cell using the following rule:

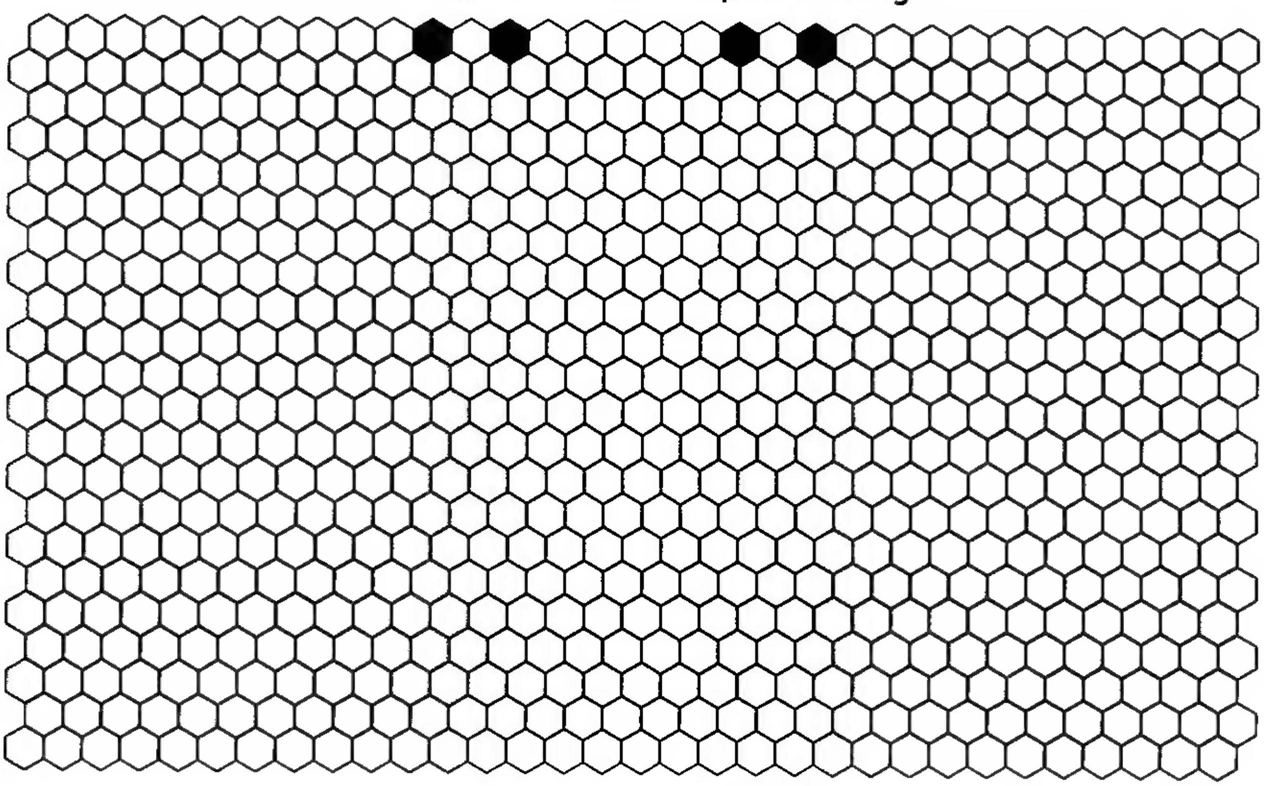
If the entry is 1, 2, 3, or 4, shade the cell **black**.
If the entry is a 0, leave the cell unshaded as **white**.

Cellular Automata

Coloring Pascal's Triangle shows an evolution process where row after row of cells grow according to specific rules: for example, coloring cells containing odd numbers black and cells containing even numbers white. Geometric patterns or structures may emerge. When the rules are changed, new structures are revealed. Processes of this type are called cellular automata. There are many applications of cellular automata, such as the simulation of fluid flow around obstacles.

This activity is designed to introduce and explore coloring look-up tables. A coloring look-up table supplies a visual definition of all the rules needed to color any particular cell based upon the colors of the cells in the row or rows immediately above it.

1. This coloring look-up table has four entries: 
   a) State verbally how it tells the coloring of a cell based on the coloring of the two cells immediately above it.
   b) Use the rule from the coloring look-up table to complete the coloring of all remaining rows in this array. What coloring pattern do you see? How is it related to Pascal's Triangle? To the Sierpinski Triangle?



2. Study the coloring of two successive rows shown from a cellular array. For each pair, construct the appropriate look-up table if the coloring is based on the two cells immediately above.

3. How many different coloring look-up tables are possible based on two cells? If the coloring of a cell is based on that of the four cells immediately above it, how many different possibilities must be shown in each look-up table?

4. Use the look-up table given to complete the coloring of this cellular array. Where have you seen the resulting structure before?